

**LEARNING MATERIAL OF
ELECTRICAL MEASUREMENT &
INSTRUMENTATION**

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MEASURING INSTRUMENTS

Static characteristics :-

- ① Accuracy
- ② Precision
- ③ Static error
- ④ Resolutions
- ⑤ Sensitivity
- ⑥ Tolerance

closeness to true values is called accuracy

① Accuracy :-
$$\frac{\text{Measured value} - \text{True value}}{\text{True value}}$$

Accuracy of percentage of full scale deflection =

$$\frac{\text{Measured value} - \text{True value}}{\text{True scale value}} \times 100$$

② Precision :- Precision of measurement is defined as the deviation of different reading from an average value.

EX: Known voltage = 100V, (101, 102, 105, 108, 105)

average = 104

Precision = $104 \pm 1\%$

③ Static error :-

$$s.e = \text{True value} - \text{measured value}$$

(static correction)

$$\delta A = \text{measured value} - \text{True value}$$

$$\delta e = \delta A$$

Precision The other parameter doesn't give 100% accurate value of temperature, but whenever the same temperature is measured it gives the same reading.

→ The thermometer is said to be a precision thermometer through it is not perfectly accurate.

→ Accuracy can be improved by calibration, but the precision can't.

$$\text{Absolute error} = A_m - A_t$$

(dA)

$$\text{Relative error} = \frac{A_m - A_t}{A_t} = \frac{\delta/A}{A_t}$$

Q. The measured value of a capacitor is ~~205.9~~ 205.9 μf where as it's true value is 201.4 μf . Determine the relative error.

$$\frac{A_m - A_t}{A_t} = \frac{3.9}{201.4} = 0.019$$

Correction: It is the algebraic difference between the true value and measured value of the quantity.

$$\text{Correction} = \text{True value} - \text{measured value}$$

$$d_c = A_t - A_m$$

$$d_c = \text{static correction}$$

$$\delta_c = -\delta A$$

Q. A meter reads 127.5V and the true value of the voltage is 127.48V. Determine the absolute error and the static correction of the instrument.

$$\begin{aligned} \delta A &= 127.5 - 127.48 \\ &= 0.02V \end{aligned}$$

$$\begin{aligned} \delta C &= 127.48 - 127.5 \\ &= -0.02V \end{aligned}$$

(iv) Sensitivity = $\frac{\text{Change in output quantity}}{\text{change in input quantity}}$

The ratio of output signal or response of the instrument to the change in input or measurement variable after the steady state has been reached it is called the sensitivity.

Q. A wheatstone bridge required a change of 7Ω in the unknown arm of the bridge to produce a change in deflection of 3mm of galvanometer. Determine the sensitivity and the deflection factor.

Ans: Sensitivity = $\frac{3}{7} \text{ mm}/\Omega = 0.43 \text{ mm}/\Omega$

Deflection factor = $\frac{1}{\text{sensitivity}}$

$$= \frac{1}{0.43} = 2.33 \Omega/\text{mm}$$

Resolution :- VRP FBI 27 2019
 When the input is slowly increased

from some arbitrary value (non zero) the output doesn't change at all until a certain increment is entered, this increment is called resolution.

OR

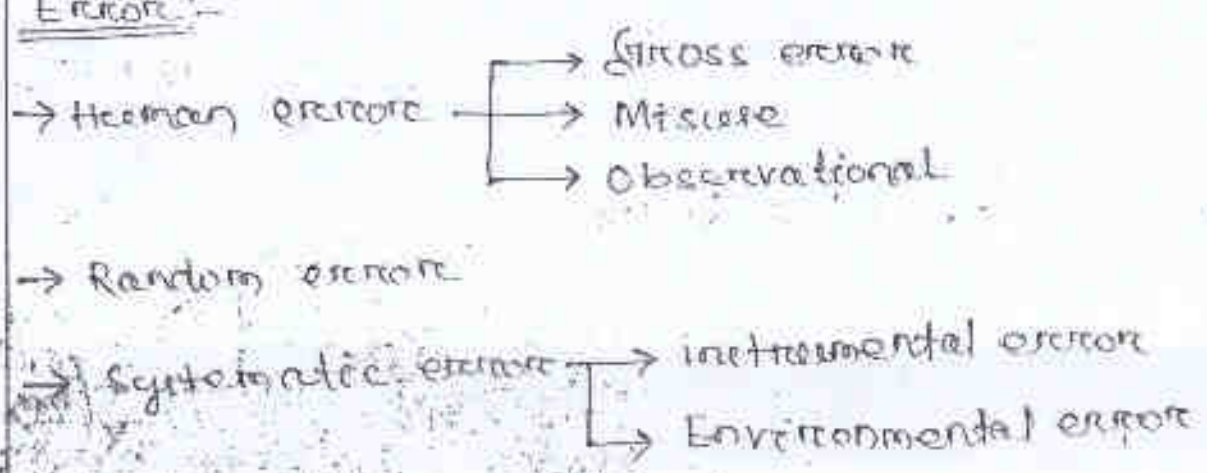
Then the smallest change in the measured value to which the instrument will respond is called the resolution.

Q. A moving coil voltmeter has a uniform scale reading is 300V and $\frac{1}{10}$ of a scale division can be estimated with a fair degree of certainty. Determine the resolution of the instrument in volt.

Ans:- 100 scale \rightarrow 300V
 1 scale \rightarrow 3V
 $\frac{1}{10}$ scale \rightarrow 0.3V

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* Errors :-



* Gross error - Gross errors are caused

from human mistakes in reading or recording values.

(Ex- Suppose an instrument shows 47.0 while the observer reads 42.0 or even if he reads the correct value records it as 41.0.

* Miscellaneous error - A casual approach on the part of the operator is the cause of this error.

* Observational - It is caused by the observer due to lack of knowledge in measurement.

All the human errors can be

avoided only by taking care in reading and recording the measurement.

* Random errors -

→ The cause such error is unknown and determined.

→ Such errors are normally small and their shows, mode of operation or probability of acceptance can be maintained hence these errors cannot be compensated.

* Systematic error

→ This type of error is caused by the system.

(a) Instrumental errors :- Instrumental errors are the errors inherent in measuring instrument because of their mechanical structure such as friction in bearings of various moving instruments.

Irregular spring tension or reduction in tension due to improper handling or overloading the instrument.

→ It may be avoided by

① Selecting a suitable instrument for particular measurement application.

② By applying correction factor after determining the amount of instrumental errors.

③ By calibrating the instrument against standardized.

(b) Environmental Errors :-

Environmental errors are due to the effects of change in temp., humidity, barometric pressure or magnetic, electrostatic field.

→ It may be avoided by

→ air conditioning

→ by use of magnetic shields

→ by use of air tied shielding in

certain components.

* Source of error - Insufficient or poor knowledge

- In sufficient knowledge of process parameters and designed condition.
- Care, designed of instruments.
- Change in process parameters, error-quality etc.
- Poor maintenance.
- Error caused by the people to operate instruments.

Electrical Measuring instruments :-

- The measurement of voltage, current and power is required to study the behaviour of an electrical equipment under certain load condition.
- Classification of various electrical instrument may be classified into two types.

① Absolute Instruments

② Secondary Instruments

① Absolute Instruments :-

Absolute instruments are those which give the value of electrical quantity to be measured in terms of physical constant and deflection only.

Ex:- Tangent galvanometer

- These instruments are extensively used

② Secondary Instruments :-

Secondary instruments are those which have been pre-calculated value with a comparison with the absolute instruments.

→ The value of the electrical quantity in these instruments can be determined from the deflection of the instruments.

→ This type of instruments are widely used.

→ Secondary instruments have been classified into three types -

① Indicating Instruments

② Recording "

③ Integrating "

① Indicating instruments :-

Indicating instruments are those which indicate the instantaneous value being measured at a particular time.

→ Their indication are given by deflection of pointers over calibrated scale.

Ex - Ammeter, voltmeter, power factor meter.

② Recording Instruments :-

Recording Instruments are those which give a continuous records of variation in the electrical quantity over a selected period of time.

Ex - Recording voltmeter.

Integrating Instruments

Integrating Instruments are those which are measured and registered by set of coils or total amount of electric energy over a selected period of the time.

Ex: Energy meter, Ampere hour meter (A-h)

* Essentials of Indicating Instruments :-

In most of the indicating instruments distinct forces are essential such as

→ Deflecting torque

→ Controlling torque

→ Damping torque

* Deflecting Torque :- It is the torque which deflects the pointer on a calibrated scale according to the electrical quantity passing through the instruments.

→ This T_d causes the moving system and hence a pointer attached to it. To restore from its zero position T_c is used. $T_d \propto I$

* Controlling Torque :- It is the torque which controls the movement of the pointer on a particular scale according to the quantity of the electricity passing through it.

→ In indicating instrument the controlling torque is obtained by two methods.

Spring control mechanism

In spring control mechanism a hair spring (of phosphorous bronze) is attached to the moving system with the deflection of the pointer the spring is twisted in the opposite direction.

- This twist in the spring produces a controlling torque which is directly proportional to the angle of deflection.
- The pointer comes to the position of rest. ($T_d = T_c$)
- The spring control torque is given

$$T_c = \frac{Ebt^3}{12L} \theta$$

E = Young's modulus constant.

b = width of the spring in kg/m^2

t = thickness of the spring in m .

L = length of the spring in m .

θ = Angle of deflection.

For a particular spring E , b , t and L are constant. So, $T_c = K\theta$

where $K = \frac{Ebt^3}{12L}$

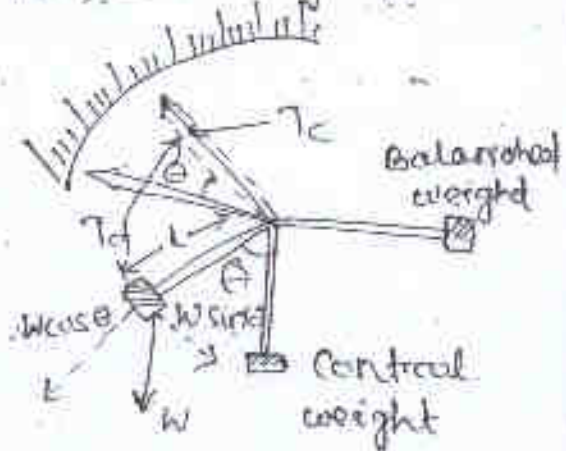
So, $T_c \propto \theta$ ($K = \text{spring constant}$)

→ In spring control system the scale is linear.

Gravity control Mechanism

Gravity control is obtained by a attaching a small weight to the moving system.

→ In such a way that
It produces a controlling torque when the system is deflected.



$$T_c = W \sin \theta = T_c = K \sin \theta$$

So, $T_c \propto \sin \theta$

In this case, scale is cramped (non-uniform)

Advantages

- It is cheap.
- It is not subjected to fatigue.
- It is not affected by temperature.

Disadvantage

- (i) Scale is non-uniform.
- (ii) The instrument has to be kept vertical.

Damping Torque

It is the torque which avoids the vibration of the pointer on a particular range of this scale.

→ There are three types of damping:

- ① Air friction damping
- ② Fluid friction damping

① Air friction damping (torsion pivots)

Air friction damping uses a thin aluminium piston or vane, which is attached to or mounted on the moving system and moves in an air chambered at one end.

② Fluid friction damping :-

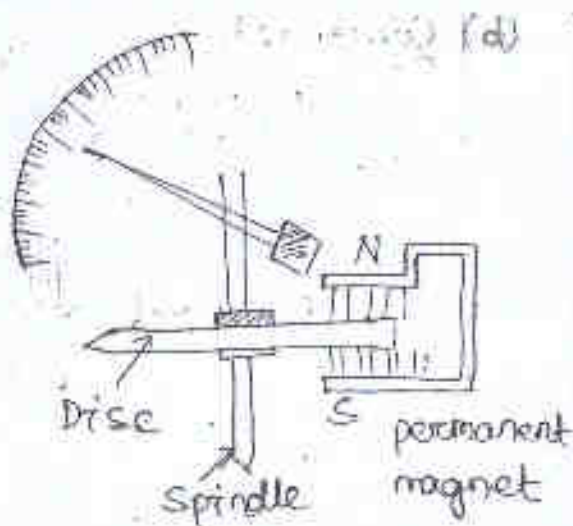
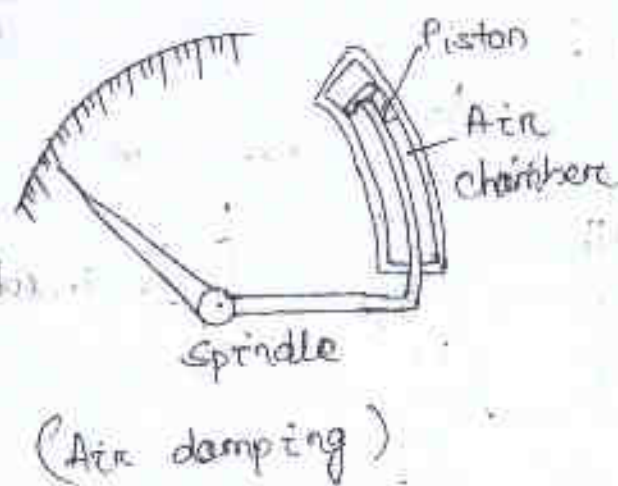
In fluid friction damping a light vane attached to the moving system & moves in an air chambered at one end is dipped in to a pot of damping oil.

- The fluid produced in assembly damping or opposing force to the vane.
- The system should be kept in vertical place.

③ Eddy current damping :-

Eddy current damping uses a conducting material which moves in magnetic field, such as to cut the lines of forces and produced eddy current.

- The torque produced by eddy current is always opposite to the original motion of the spindle (According to Lenz's Law).
- This is the most efficient type of damping and is widely used in PMMC, hot wire induction type instrument.



* Measuring Instruments :-

→ Absolute instruments

→ Secondary instruments

→ Analog instruments

→ Digital instruments

→ Analog instruments

- Indicating
- Recording
- Integrating

② ANALOG AMMETER AND VOLT METER :-

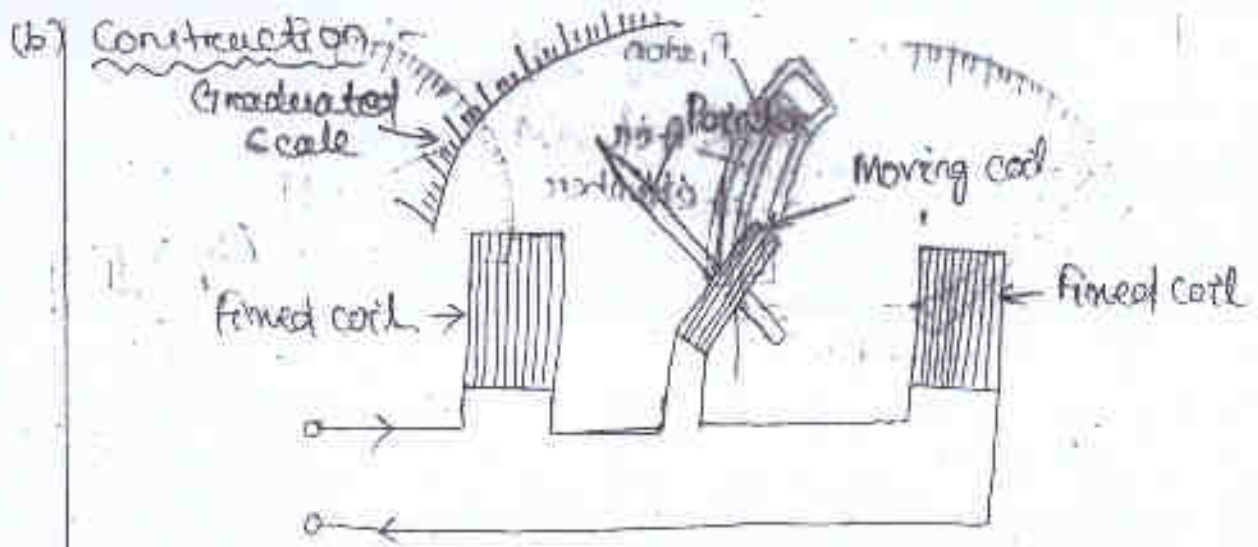
* Moving coil type Instrument (M-C) :-

In these type of indicating instruments

the pointer is attached to a coil which moves over a calibrated scale.

→ The operating current is allowed to flow in the moving coil which produces the necessary deflecting torque. The moving coil type instruments are classified as:-

(a) Permanent magnet moving coil (PMMC)



* Fined coil :- The field is produced by the fined coil. The coil is divided into two sections to give a more uniform field near the centre and to allow passage of the instrument shaft.

→ The coil wound with fine wire for measuring low currents and with thick wire for measuring higher currents.

→ The wire is stranded in order to reduce the eddy current losses in the conductor.

* Moving coil :- The moving coil is wound either as a self supporting coil or else on a non-metallic former.

* Control system :-

Controlling torque is produced by

two control springs. These springs act as leads to the moving system.

* Damping system. →

in these instruments.

→ And is provided by a pair of aluminium vanes attached to the spindle at the bottom. These vanes move in sector shaped chambers. Eddy current damping cannot be used in these instruments as the operating field is very weak (on account of the fact that the coil are air core) and any introduction of a permanent magnet required for eddy current damping would distort the operating magnetic field of the instrument.

* Torque eqⁿ :-

Let r_1 = instantaneous value of current in the fixed coil.

r_2 = Instantaneous value of current in the moving coil.

L_1 = self-inductance of fixed coil.

L_2 = self-inductance of moving coil.

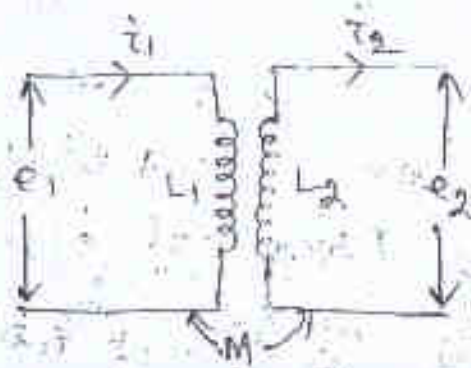
M = Mutual inductance between fixed and moving coil.

Flux linkage in fixed coil

$$\Phi_1 = L_1 i_1 + M i_2$$

Flux linkage in moving coil

$$\Phi_2 = L_2 i_2 + M i_1$$



Electrical input energy

$$= e_1 i_1 dt + e_2 i_2 dt = i_1 d\phi_1 + i_2 d\phi_2$$

$$\text{as } e_1 = \frac{d\phi_1}{dt} \text{ and } e_2 = \frac{d\phi_2}{dt}$$

$$= i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_1)$$

$$= d(L_1 i_1 + M i_2) i_1 + d(L_2 i_2 + M i_1) i_2$$

$$= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 M di_2 + i_1 i_2 dM + i_2^2 dL_2 + i_2 L_2 di_2 + i_2 M di_1 + i_1 i_2 dM$$

$$= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 M di_2 + 2 i_1 i_2 dM + i_2^2 dL_2 + i_2 L_2 di_2 + i_2 M di_1$$

→ Electrical stored energy →

$$= \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$

→ Change in electrical stored energy :-

$$d\left(\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2\right)$$

$$= \frac{1}{2} (L_1 \times 2 i_1) + L_1 i_2 di_2 + M i_1 di_2 + M i_2 di_1 + i_1 i_2 dM$$

$$= L_1 i_1 di_1 + L_1 i_2 di_2$$

$$= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1 \quad \text{--- (1)}$$

Electrical stored energy

$$= \frac{1}{2} i_1^2 L_1 + \frac{1}{2} i_2^2 L_2 + i_1 i_2 M$$

$$= \frac{1}{2} (L_1 \times 2 i_1 di_1 + L_2 \times 2 i_2 di_2) + M i_1 di_2 + M i_2 di_1$$

Total electrical input energy = change in stored energy + mechanical energy

$$M \cdot E = T \cdot E \cdot E - \Delta E \cdot E$$

$$= \tau_1 \tau_2 dM + \frac{1}{2} \tau_1^2 dL_1 + \frac{1}{2} \tau_2^2 dL_2$$

$$= \tau_1 \tau_2 dM$$

(as L_1 & L_2 are constant)

$$\text{So, } \frac{1}{2} \tau_1^2 dL_1 = 0$$

$$\frac{1}{2} \tau_2^2 dL_2 = 0$$

$$\Rightarrow T_d \times d\theta = \tau_1 \tau_2 dM$$

$$\Rightarrow T_d = \tau_1 \tau_2 \frac{dM}{d\theta}$$

In case of D.C.

$$T_d = 2T I_2 \frac{dM}{d\theta}$$

In case of A.C.

$$I_1 = I_{m1} \sin \omega t, I_2 = I_{m2} \sin(\omega t - \phi)$$

$$T_d = \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T \tau_1 \tau_2 dt$$

$$= \frac{dM}{d\theta} \cdot \frac{1}{2\pi} \int_0^{2\pi} I_{m1} \sin \omega t \cdot I_{m2} \sin(\omega t - \phi) d\omega t$$

$$= \frac{dM}{d\theta} \cdot \frac{I_{m1} I_{m2}}{2\pi} \int_0^{2\pi} 2 \sin \omega t \cdot \sin(\omega t - \phi) d\omega t$$

$$= \frac{dM}{d\theta} \cdot \frac{I_{m1} I_{m2}}{2\pi} \int_0^{2\pi} \cos(\omega t - \omega t + \phi) - \cos(2\omega t - \phi) d\omega t$$

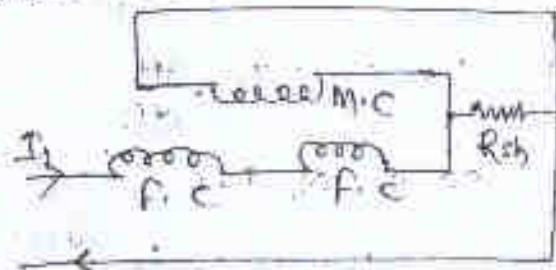
$$= \frac{dM}{d\theta} \cdot \frac{I_{m1} I_{m2}}{4\pi} \int_0^{2\pi} \cos \phi - \cos(2\omega t - \phi) d\omega t$$

$$= \frac{dM}{d\theta} \cdot \frac{I_{m1} I_{m2}}{4\pi} \left\{ \left[\cos \phi \cdot \omega t \right]_0^{2\pi} - \left[\frac{\sin(2\omega t - \phi)}{2} \right]_0^{2\pi} \right\}$$

$$\begin{aligned}
 &= \frac{dM}{d\theta} \cdot \frac{I_1 I_2}{2\pi} \left[\cos \phi (2\pi - \theta) \right] - \frac{1}{2} \left[\sin^2 (\frac{\theta}{2}) - \sin^2 \phi \right] \\
 &= \frac{dM}{d\theta} \cdot \frac{I_1 I_2}{2\pi} \cdot 2\pi \cos \phi \\
 &= \frac{dM}{d\theta} \cdot \frac{I_1 I_2}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cos \phi \\
 &= I_1 I_2 \cos \phi \frac{dM}{d\theta}
 \end{aligned}$$

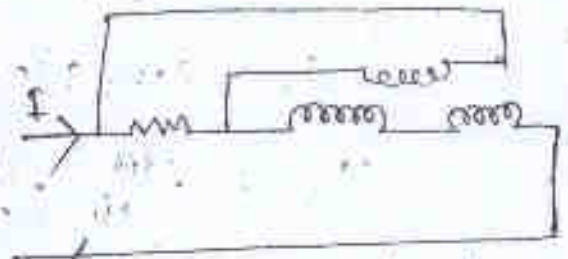
* Dynamometer as ammeter -

In the fig. shows that the arrangement of coil of an electro-dynamometer ammeter



(Ammeter of small range)

→ In this case the fixed coil and moving coil are connected in series and



(Ammeter of high range)

It carries the same current i.e. $I_1 = I_2 = I$ and $\phi = 0$

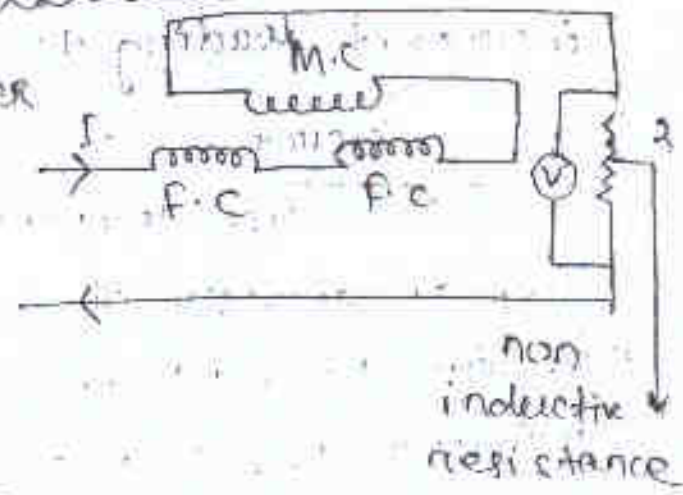
Deflecting torque

$$T_d = I^2 \frac{dM}{d\theta}$$

$$\theta = \frac{I^2}{K} \frac{dM}{d\theta} \quad (\because T_d = K\theta)$$

Electrodynamometer as a Voltmeter

The electrodynamic movement is used as a Volt meter by connecting the fixed and moving coil in series with a high non-inductive resistance.



$$T_d = I_1 I_2 \cos \phi \frac{dM}{d\theta}$$

$$I_1 = I_2 = \frac{V}{Z} \text{ and } \phi = 0$$

$$T_d = \left(\frac{V}{Z}\right) \left(\frac{V}{Z}\right) \frac{dM}{d\theta}$$

$$T_d = K \theta$$

At equilibrium, $T_d = T_c$

$$\frac{V^2}{Z^2} \frac{dM}{d\theta} = K \theta$$

$$\theta = \frac{V^2}{K Z^2} \frac{dM}{d\theta}$$

Errors:-

Fractional Error:-

→ The magnetic field produced by air core is low. Hence, to produce high magnetic field either the current increased or the no. of turns can be increased.

→ The increase of current will produce heat. Increased the no. of turns of

as a result the weight of the system is reduced, leading to a frictional error.

Temperature Error:-

The temperature error is generally seen in an ammeter of large range due to flow of high current. To reduce this escaping resistance must be connected in series with the meter.

* Error due to stray magnetic field:-

Generally the field in the system is very weak so it is greatly affected by the strong magnetic field present outside.

→ This can be reduced by using a magnetic shield.

* Merits of Dynamometer type instruments:-

- (i) It can be used for both A.C. and D.C.
- (ii) These instruments are free from hysteresis & eddy current loss.
- (iii) Dynamometer type volt meter are very useful for accurate measurement of rms value of voltage irrespective of wave form.

* Demerits:-

- (i) The scale is non-uniform.
- (ii) Friction loss is more than other instrument.
- (iii) Power consumption is comparatively high.

Moving iron Instrument

Principle of working:— A plate or vane

of soft iron of high permeability steel form

the moving element of the system. The iron vane is situated in such a manner that it

can move in a magnetic field produced by a stationary coil. The coil is excited by the current or voltage under measurement.

When the coil is excited it becomes an

electromagnet and the iron vane moves in

such a way so as to increase the flux of

the electromagnet.

* Torque eqⁿ:—

Let 'I' be the current under measurement the emf induced in the coil is,

$$e = \frac{d\phi}{dt} \quad (\text{where } \phi = LI)$$

$$\frac{d(LI)}{dt} = L \frac{di}{dt} + i \frac{dL}{dt}$$

Input electrical energy = $e i dt$

$$= \left[L \frac{di}{dt} + i \frac{dL}{dt} \right] I dt$$

$$= LI dI + I^2 dt$$

Energy stored in the coil = $\frac{1}{2} LI^2$

Energy stored in a later increment

$$= \frac{1}{2} (L + dL) (I + dI)^2$$

change in stored energy = ...

$$\frac{1}{2} (L+dL) (I+dI)^2 - \frac{1}{2} LI^2$$

$$= \frac{1}{2} (L+dL) (I^2 + dI^2 + 2IdI) - \frac{1}{2} LI^2$$

$$= \frac{1}{2} LI^2 + \frac{1}{2} LdI^2 + ILdI + \frac{1}{2} I^2 dL + dI^2 \cdot \frac{dL}{2} + I \cdot dLdI - \frac{1}{2} LI^2$$

(neglecting the 2nd order & higher order we will get -

$$\text{change in stored energy} = ILdI + \frac{1}{2} I^2 dL$$

Electrical input Energy = change in stored energy + Mechanical energy

$$LIdI + I^2 dL = ILdI + \frac{1}{2} I^2 dL + T_d d\theta$$

$$\Rightarrow T_d d\theta = \frac{1}{2} I^2 dL$$

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}, \quad T_c = K\theta$$

At equilibrium, $T_d = T_c$

$$\Rightarrow K\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\theta = \frac{1}{2} \frac{I^2}{K} \frac{dL}{d\theta}$$

Hence the deflecting torque is proportional to the square of the rms value. Therefore the deflecting torque is unidirectional. The direction shall be the polarity of the

current from the scale is non-centrism, if the change of inductance with angle of deflection is constant but in actual practise

$\left(\frac{dL}{d\theta}\right)$ is not constant.

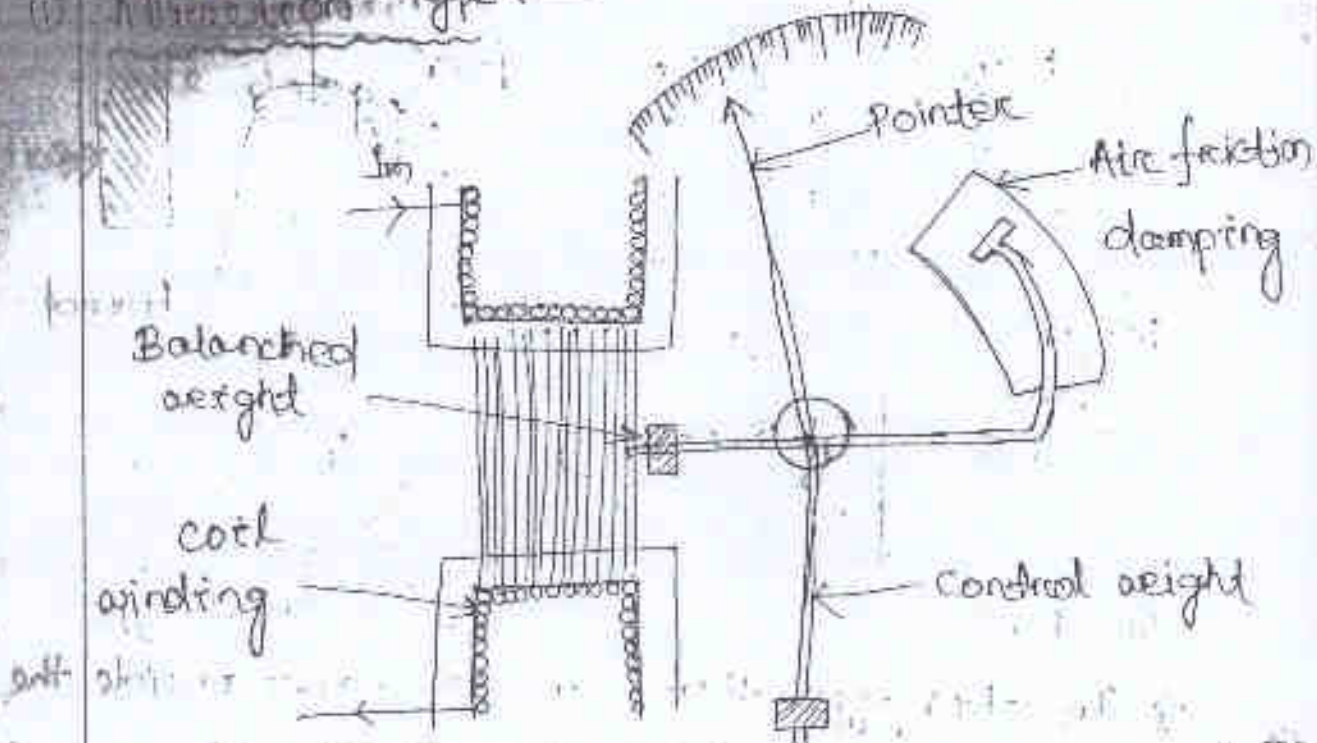
Hence it is possible to design and construct a centric scale over a considerable part of its length.

Classification of M.I instrument

It is of two types -

- 1. Attraction type
- 2. Repulsion type

1. Attraction type :-



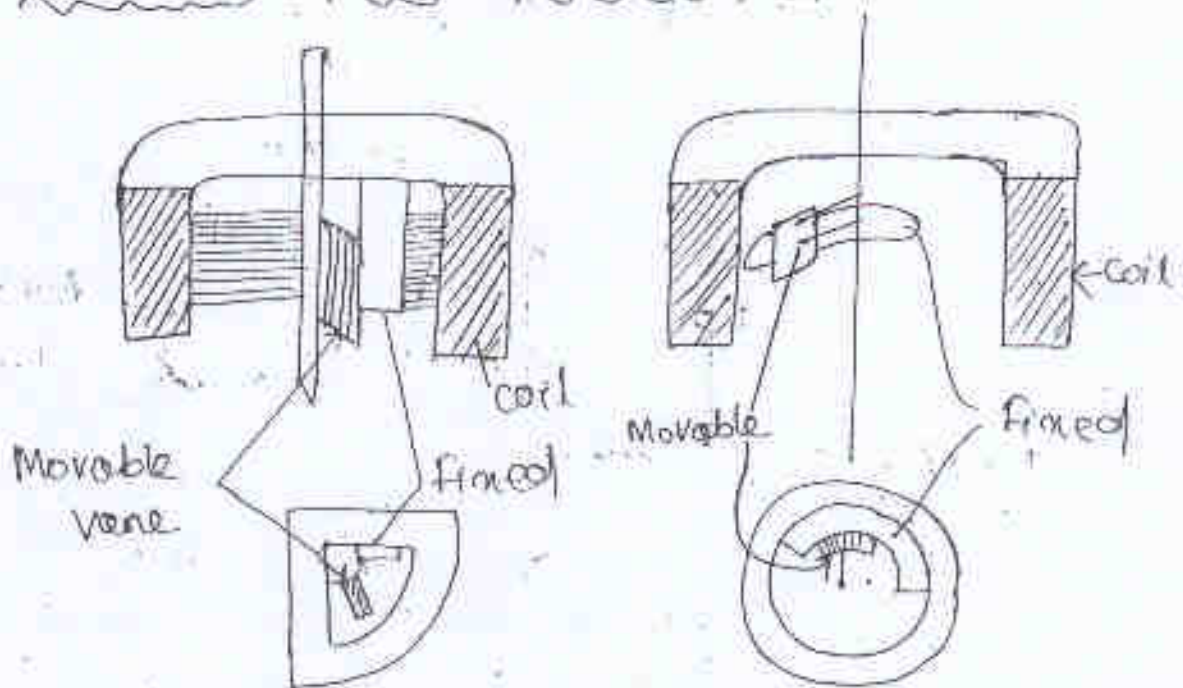
In the type above fig. the moving iron is a flat disc or a sector concentrically mounted.

→ The coil is flat and has a narrow slot

1. Attraction type When the current flows through the coil a magnetic field is produced and the moving iron moves from the weaker field outside the coil to the stronger field inside it.

- The controlling torque is provided by spring controlling system or gravity controlling system in case of vertically mounted instrument.
- The damping is produced by the air friction damping.

② Repulsion type M.I. instrument →



(a) Radial type

(b) Co-axial type

→ In this type there are two vanes inside the coil, one fixed and another movable.

→ These two vanes are similarly magnetised when the current flows through the coil & there is a force of repulsion between the two vanes resulting in the

According to the construction these instruments are of two types:-

Radial ^{vane} type:- In this type the vanes are radial strips of iron. The strips are placed within the coil. The fixed vane is attached to the coil and the movable one to the spindle of the instrument.

Co-axial vane type:- In this type of instrument the fixed and moving vanes are section of co-axial cylinders.

The controlling torque is provided by spring in vertically mounted instrument gravity control can be applied.

Over direction damping is provided to produce self damping torque.

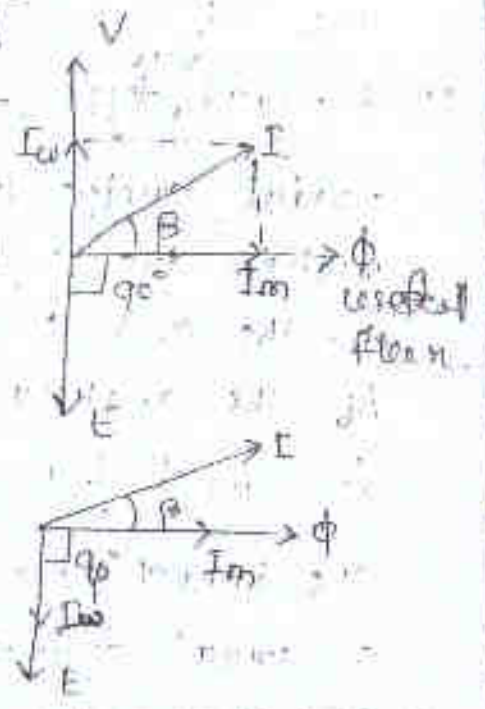
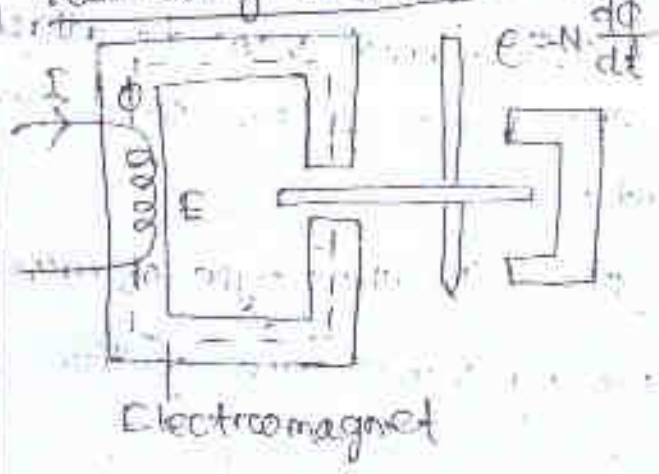
In moving iron instrument the operating magnetic field is weak and therefore eddy current damping is not used. whatever may be the direction of current in the coil of the instruments. The iron vanes are so magnetised that there is always a force of attraction in attraction type and the repulsion in the repulsion type instrument. Therefore these instruments can be used both for A.C & D.C.

Errors are hysteresis, temperature errors, stray magnetic field in case of both A.C and D.C

Frequency error is seen in only A.C.

Induction Type instrument

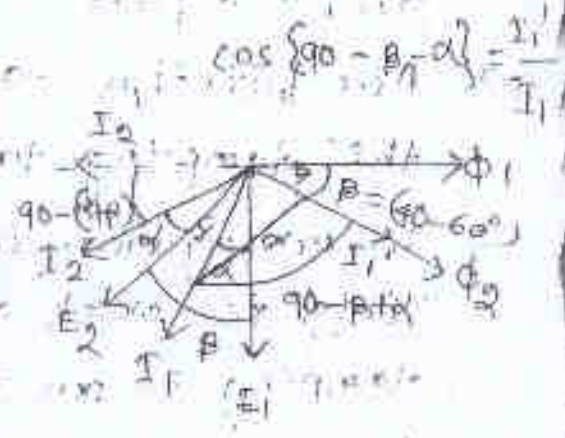
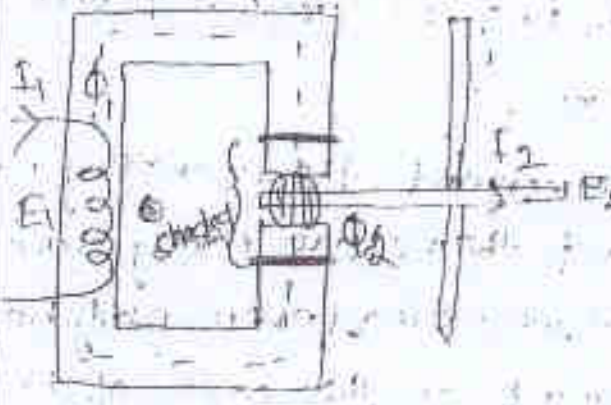
A.C. Measurement →
Retarding method



→ Torque = $\phi \text{ len} \times \text{Component}$

⇒ $T \propto \phi I$

unshaded portion



Torque eqn :-

$T_{\text{tot}} = K (\phi_2 I_1 + \phi_1 I_2)$

$T = K [\phi_2 I_1 \cos(90 - (\beta - \alpha)) - \phi_1 I_2 \cos(90 - (\beta + \alpha))]$

$T = K [\phi_2 I_1 \sin(\beta - \alpha) - \phi_1 I_2 \sin(\beta + \alpha)]$

$T = K [\phi_2 \frac{E_1}{Z} \sin(\beta - \alpha) - \phi_1 I_2 \sin(\beta + \alpha)]$

$(I_1 = \frac{E_1}{Z}, I_2 = \frac{E_2}{Z})$

$E_1 \propto F \phi_1$
 $E_2 \propto F \phi_2$
(Induction only AC in $F \propto i$)

$$A_{\text{total}} = K \left[\Phi_2 \frac{F \Phi_1}{Z} \sin(\beta - \alpha) + \Phi_1 \frac{F \Phi_2}{Z} \sin(\beta + \alpha) \right]$$

$$= K \frac{F \Phi_1 \Phi_2}{Z} \left[\sin(\beta - \alpha) + \sin(\beta + \alpha) \right]$$

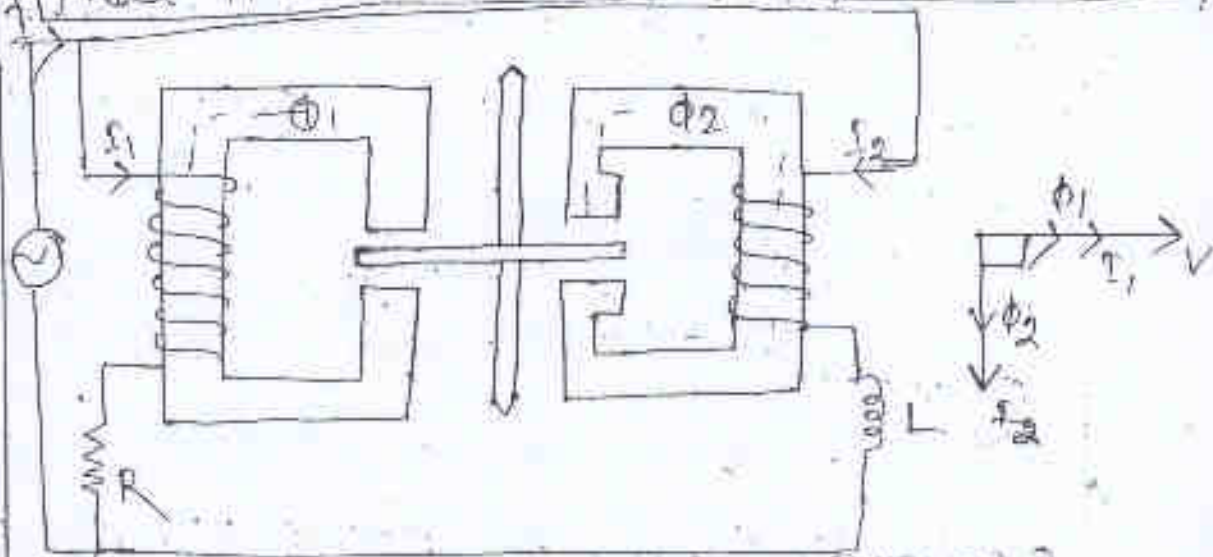
$$\therefore E_1 \propto F \Phi_1, \\ E_2 \propto F \Phi_2$$

$$= \frac{2 K F \Phi_1 \Phi_2}{Z} (\cos \alpha \sin \beta)$$

$$= \frac{K' F \Phi_1 \Phi_2}{Z} (\cos \alpha \sin \beta) \quad (2K = K')$$

$$T_d = \frac{K' F I^2}{Z} (\cos \alpha \sin \beta)$$

when the value of $\alpha = 0^\circ$ and $\beta = 90^\circ$ at that time the term is maximum.

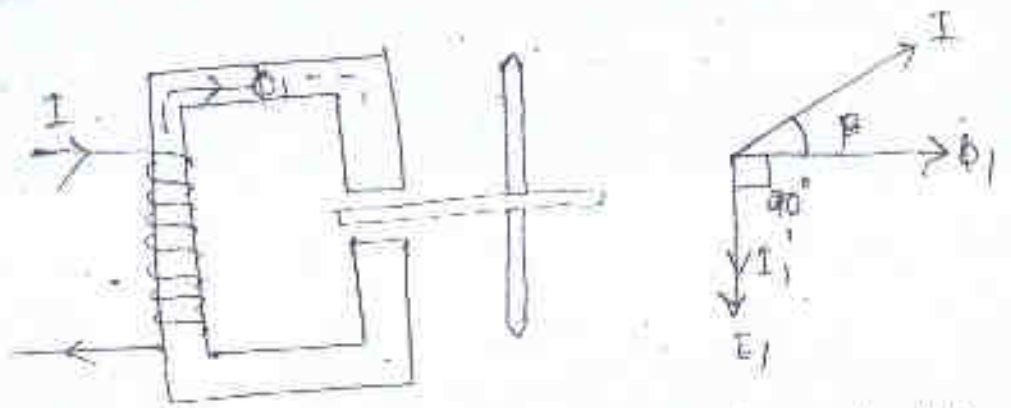


(Two pole method)

Induction type instruments:- These instruments are suitable for A.C. measurements only. Here the T_d is produced by the eddy currents induced in an aluminium disc by the flux created by an electromagnet.

Working principle :-

07/01/2015



→ The operation of all induction instruments depends on the production of torque due to the reaction between a flux and eddy currents induced in a metal disc by another flux.

→ Since the magnitude of eddy currents also depends on the flux producing them. The instantaneous value of torque is proportional to the square of current or voltage under measurement.

$$T_d = k \phi_2 I_1' - \phi_1 I_2'$$

$$= I_2 I_1' \text{ (Square)}$$

Operation :-

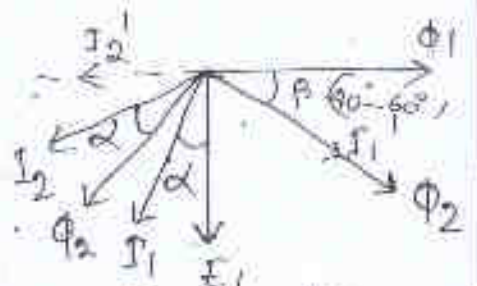
→ Consider a thin aluminium disc (copper disc) which is placed between the two edges of electromagnet as given in the above fig.

→ Since aluminium disc acts as a secondary of the transformer, an emf (E) which is lagging behind the flux ϕ by $\frac{\pi}{2}$ rad. is induced in it.

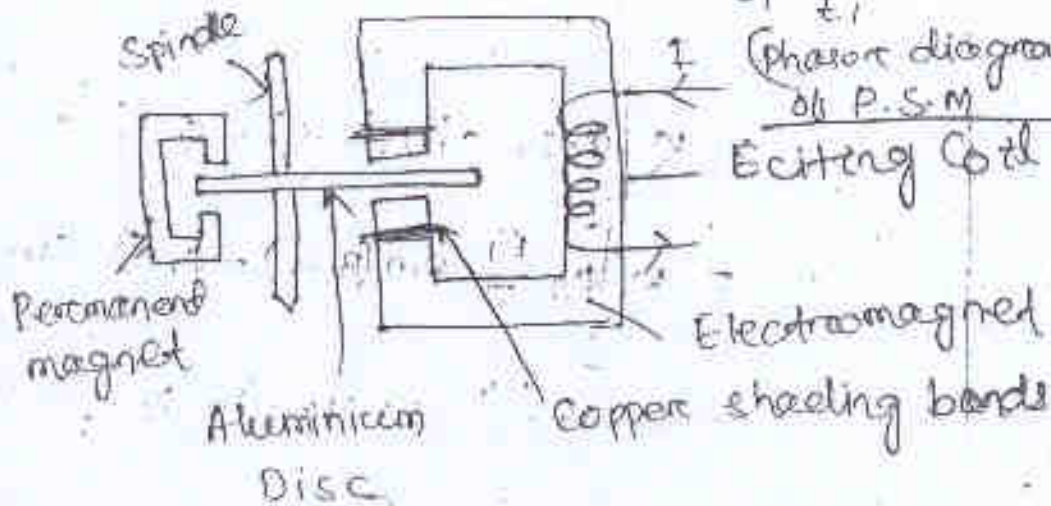
As the disc is pure resistive therefore all current I' with lag behind the main flux ϕ by $\frac{\pi}{2}$ radians. As the component of I' along ϕ is zero, so, torque produced is 0.

Hence to obtain resulting torque it is necessary to produce an eddy current which is either less or more, than $\frac{\pi}{2}$ radian.

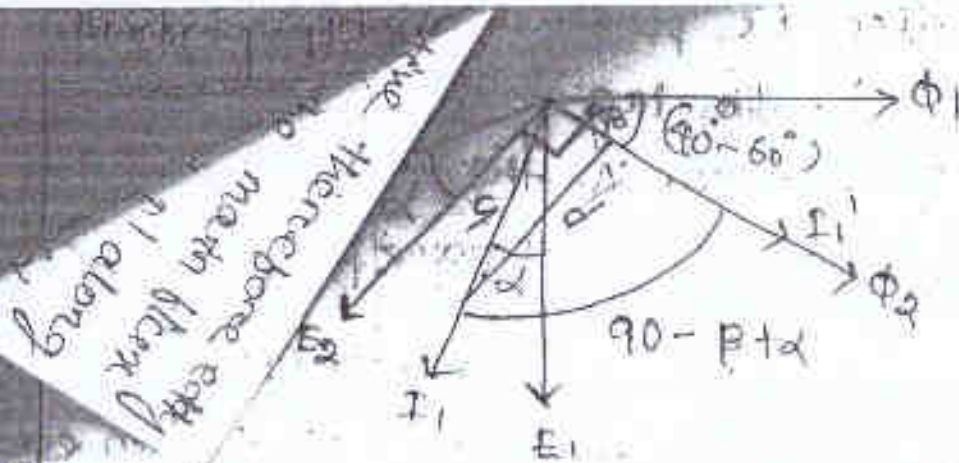
Pole shading Method :-



(Phasor diagram) of P.S.M



In this method working current is passed through the coil electromagnet. Permanent magnet is used for damping. Controlling torque is provided by spring. Half of each pole face is surrounded by a copper band in order to split the working flux.



Torque eqn :-

$$T_d = K(\phi_2 I_1' - \phi_1 I_2')$$

$$= K \left[\phi_2 I_1' \cos 90^\circ - (\beta - \alpha) - \phi_1 I_2' \cos 90^\circ (\alpha + \beta) \right]$$

$$= K \left[\phi_2 I_1' \sin(\beta - \alpha) - \phi_1 I_2' \sin(\alpha + \beta) \right]$$

$$= K \left[\phi_2 \frac{E_1}{Z} \sin(\beta - \alpha) - \phi_1 \frac{E_2}{Z} \sin(\alpha + \beta) \right]$$

$$= K \left[\phi_2 \frac{F \phi_1}{Z} \sin(\beta - \alpha) - \phi_1 \frac{F \phi_2}{Z} \sin(\alpha + \beta) \right]$$

$$= K \frac{F \phi_1 \phi_2}{Z} \left[\sin(\beta - \alpha) - \sin(\alpha + \beta) \right]$$

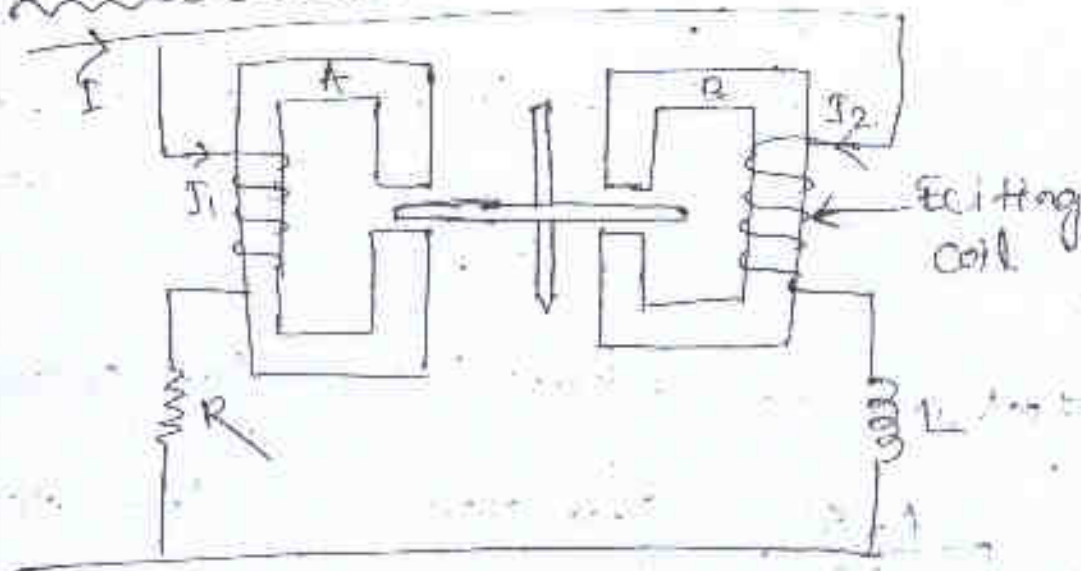
$$= \frac{2KF \phi_1 \phi_2}{Z} (\cos \alpha \cdot \sin \beta)$$

$$= \frac{K' F \phi_1 \phi_2}{Z} (\cos \alpha \cdot \sin \beta)$$

$$T_d = \frac{K' F \phi_1 \phi_2}{Z} (\cos \alpha \cdot \sin \beta)$$

From the above ~~equations~~ it can be seen that max^m torque will be developed when $d=0$ and $\beta = 90^\circ$.

Two pole method :-



In this method a non-inductive resistance R connected in series with the magnetizing coil of magnet 'A' and a inductive coil 'L' is connected in series with the magnetizing coil of magnet 'B'.

Thus there will be two magnetic field having a phase difference of nearly 90° , which will produce a resultant torque on the disc.

Advantages :-

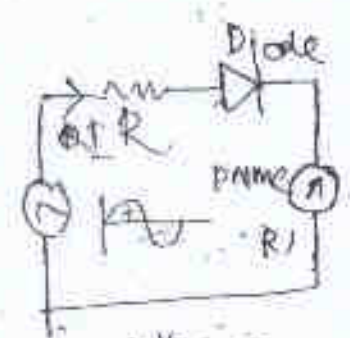
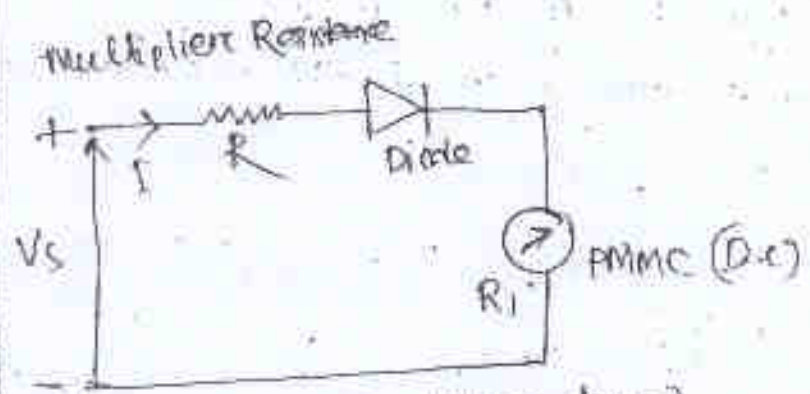
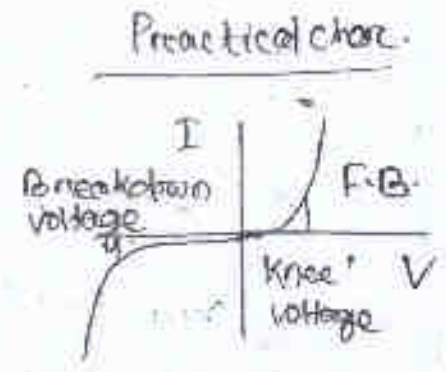
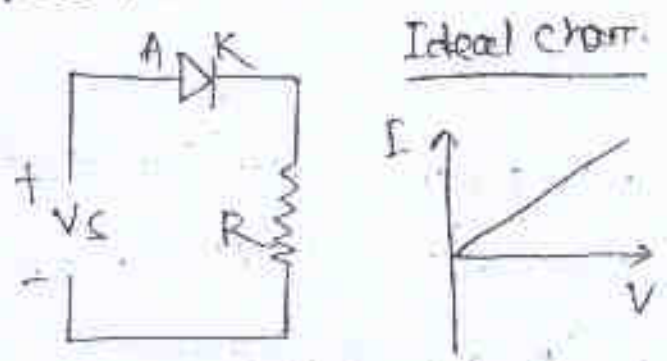
- (i) A full scale deflection of 90° can be obtained.
- (ii) Effect of stray magnetic field is small.
- (iii) Damping is easier and effective.

* Disadvantages :-

- (i) These instruments are costly and consumed more power.
- (ii) Greater deflection causes more stresses in the control speed.
- (iii) Variation in supply frequency and temperature may cause serious errors unless compensating device is employed.

* Rectifier type instrument :-

Date 08/01/15



(Half wave Rectifier type)

In case D.C. \rightarrow

$$V_{dc} = I(R_i + R)$$

$$\Rightarrow I = \frac{V_{dc}}{(R_i + R)} = \frac{V_{RMS}}{(R_i + R)}$$

$\frac{V_m}{\sqrt{2}}$ Rectifying D.C.

In case of A.C.

$$I = \frac{V_{ac}}{R_i + R} = \frac{V_m \sin \omega t}{R_i + R}$$

$$\frac{V_{avg}}{I} = \frac{V_m}{(R_1 + R_2)} \frac{1}{2\pi} \int_0^{2\pi} \sin \omega t \, d\omega t$$

$$= \frac{-V_m}{(R_1 + R_2)} \times \frac{1}{2\pi} [-\cos \omega t]_0^{2\pi}$$

$$= \frac{V_m}{R_1 + R_2} \times \frac{1}{2\pi} [\cos \pi - \cos 0]$$

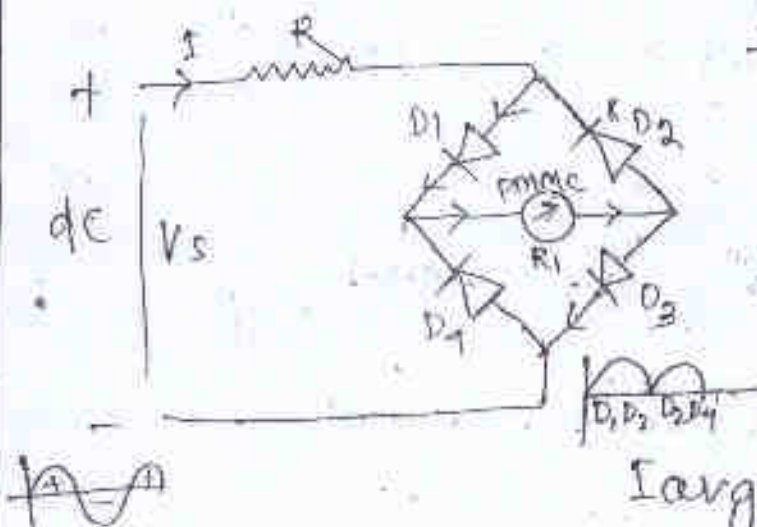
$$= \frac{V_m}{R_1 + R_2} \times \frac{2}{2\pi}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$= \frac{V_m}{\pi (R_1 + R_2)} = \frac{\sqrt{2} V_{rms}}{\pi (R_1 + R_2)}$$

$$= \frac{0.45 V_{rms}}{(R_1 + R_2)}$$

∴ Power form factor = $\frac{V_{rms}}{0.45 V_{rms}} = \frac{1}{0.45} = 2.22$



In case D.C

$$I_{dc} = \frac{V_{dc}}{R_1 + R_2}$$

In case of AC

$$I_{ac} = \frac{V_{ac}}{R_1 + R_2}$$

$$I_{avg} = \frac{V_{ac}}{R_1 + R_2} = \frac{V_m \sin \omega t}{R_1 + R_2}$$

$$V_{avg} = \frac{1}{\pi} \int_0^{\pi} V_m \sin \omega t \, d\omega t$$

$$= \frac{-V_m}{\pi} [\cos \omega t]_0^{\pi}$$

$$= \frac{2}{\pi} V_m$$

$$X_L = 2\pi f \times 0.6 = 188.4 \Omega$$

$$\Rightarrow V - V_m = 600 - 120 = 480V$$

$$R_{se} = 2400 \Omega$$

$$\frac{V_m}{I_m} = \sqrt{R_m^2 + X_L^2} = 2407.38 \Omega$$

$$\Rightarrow I_m = \frac{V_m}{\sqrt{R_m^2 + X_L^2}} = \frac{120}{\sqrt{(2407)^2 + (188.4)^2}} = 0.05A$$

$$V = I_m \sqrt{(R_m + R_{meas})^2 + (X_L)^2}$$

$$\left(\frac{600}{0.049}\right)^2 = (R_m + R_{meas})^2 + (X_L)^2$$

$$(R_m + R_{se}) = \sqrt{Z^2 - X_L^2}$$

$$R_{se} = \sqrt{Z^2 - (X_L)^2} - R_m$$

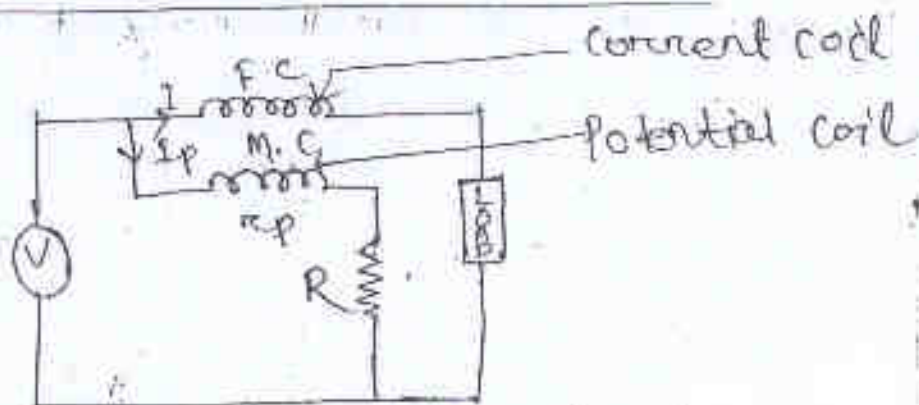
$$= \sqrt{(12244)^2 - (188.4)^2} - 2400$$

$$= 12242.55 - 2400$$

$$= 9842.5 \Omega$$

③ WATTMETERS AND MEASUREMENT OF POWER

* E. Dynamometer as a watt meter:

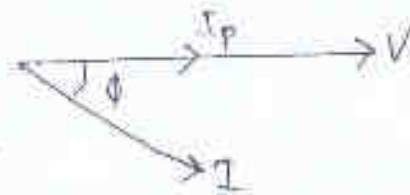


V = Voltage across potential coil

I_p = Current in potential coil

I = Current in current coil

Phasor diagram



$$\phi' = \phi - \beta$$

$$T_d = I_p I \cdot \cos \phi' \frac{dM}{d\theta}$$

$$= \frac{V}{R_p} I \cdot \cos(\phi - \beta) dM$$

$$= \frac{V}{Z_p} I \cdot \cos(\phi - \beta)$$

$$= \frac{V}{(R_p + jX_p)^2} I \cdot \cos(\phi - \beta) \frac{dM}{d\theta}$$

$$= \frac{V}{R_p / \cos \beta} I \cos(\phi - \beta) \frac{dM}{d\theta}$$

$$= \frac{V}{R_p} I \cos \beta \cdot \cos(\phi - \beta) \frac{dM}{d\theta}$$

$$= \frac{\text{True reading}}{\text{Actual reading}} = \frac{\frac{V}{R_p} I \cos \phi \frac{dM}{d\theta}}{\frac{V}{R_p} I \cos \beta \cdot \cos(\phi - \beta) \frac{dM}{d\theta}}$$

$$= \boxed{\frac{\cos \phi}{\cos \beta \cdot \cos(\phi - \beta)}}$$

$$\text{True value} = \frac{\cos \phi}{\cos \beta \cos(\phi - \beta)} \times \text{Actual value}$$

$$\Rightarrow \text{Error} = \text{Actual value} - \frac{\cos \phi}{\cos \beta \cos(\phi - \beta)} \times \text{Actual value}$$

$$\text{Actual value} \left[1 - \frac{\cos \phi}{\cos \beta \cos(\phi - \beta)} \right]$$

$$= \text{Actual} \cdot \left(\frac{\cos \phi \cdot \cos(\phi - \beta) - \cos \phi}{\cos \beta \cdot (\cos \phi - \beta)} \right)$$

$$= \text{Actual} \left[\frac{\cos \phi \cdot \cos \beta + \sin \phi \cdot \sin \beta - \cos \phi}{\cos \phi \cdot \cos \beta + \sin \phi \cdot \sin \beta} \right]$$

$$= \text{Actual} \left[\frac{\cos \phi + \sin \beta - \cos \phi}{\cos \phi + \sin \beta} \right] \begin{matrix} (\cos \beta = 1) \\ (\sin \phi = 1) \end{matrix}$$

$$= \text{Actual value} \times \left(\frac{\sin \beta}{\cos \phi + \sin \beta} \right)$$

$$\frac{\text{True}}{\text{Actual}} = \frac{\cos \phi}{(\cos \beta \cdot \cos \phi + \sin \phi \cdot \sin \beta)}$$

$$= \frac{\cos \phi}{\cos^2 \beta + \sin \phi \cdot \sin \beta \cdot \cos \beta}$$

$$= \frac{\cos \phi / \cos^2 \beta}{\frac{\cos \phi \cos^2 \beta}{\cos^2 \beta} + \frac{\sin \phi \cdot \sin \beta \cdot \cos \beta}{\cos^2 \beta}}$$

$$= \frac{\cos \phi / \cos^2 \beta}{\cos \phi + \frac{\sin \phi \cdot \sin \beta}{\cos \beta}}$$

$$= \frac{\cos \phi \cdot \sec^2 \beta}{\cos \phi + \sin \phi \cdot \tan \beta}$$

$$= \frac{\cos \phi (\sec^2 \beta)}{\cos \phi \left(1 + \frac{\sin \phi \cdot \tan \beta}{\cos \phi} \right)}$$

$$= \frac{\sec^2 \beta}{1 + \frac{\sin \phi \cdot \tan \beta}{\cos \phi}}$$

$$= \frac{1 + \tan^2 \phi \tan^2 \beta}{1 + \tan \phi \tan \beta}$$

$$= \frac{1}{1 + \tan \phi \tan \beta} \quad (\tan^2 \beta \ll \ll \ll 1)$$

Error = Actual value - true value

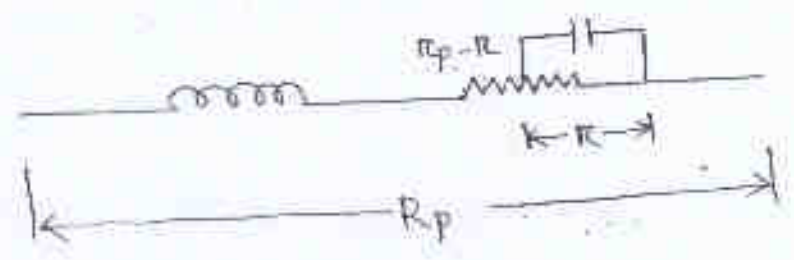
$$= \text{Actual value} - \frac{1}{1 + \tan \phi \tan \beta} \times \text{Actual value}$$

$$= \text{Actual value} \left(1 - \frac{1}{1 + \tan \phi \tan \beta} \right)$$

$$= \text{Actual value} \left[\frac{X + \tan \phi \tan \beta - X}{1 + \tan \phi \tan \beta} \right]$$

$$= \text{Actual value} \left[\frac{\tan \phi \tan \beta}{1 + \tan \phi \tan \beta} \right]$$

$$= \text{Actual value} \left[\tan(\phi + \beta) \right]$$



$$Z_p = \sqrt{(R_p)^2 + (X_L)^2}$$

$$= j\omega L + (R_p - \pi) + \frac{\pi \times \frac{-j}{\omega C}}{\pi + \left(\frac{j}{\omega C}\right)}$$

$$= j\omega L + (R_p - \pi) + \left(\frac{(-1)\pi j}{\pi \omega C (-1)j} \right)$$

$$= j\omega L + (R_p - \pi) + \frac{\pi(-j)(\omega \pi C + j)}{\dots}$$

$$= j\omega L + (R_p - r) + \frac{r(-j)(\omega r p + j \omega r c)}{(\omega r c)^2 - (j)^2}$$

$$= j\omega L + (R_p - r) + \frac{-j r^2 \omega c + r c}{\omega r c^2 + 1}$$

$$= j\omega L + (R_p - r) + \frac{r - j \omega r^2 c}{\omega^2 c^2 r^2 + 1}$$

$$= j\omega L + R_p - r + (r - j \omega r^2 c)$$

$$= j\omega L + R_p - j \omega r^2 c$$

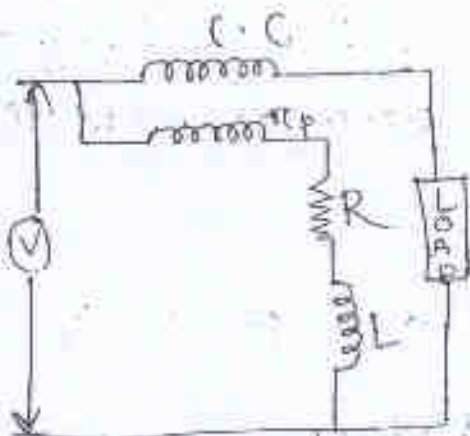
$$= j\omega (L - r^2 c) + R_p$$

when $L = r^2 c$ $Z_p = R_p$

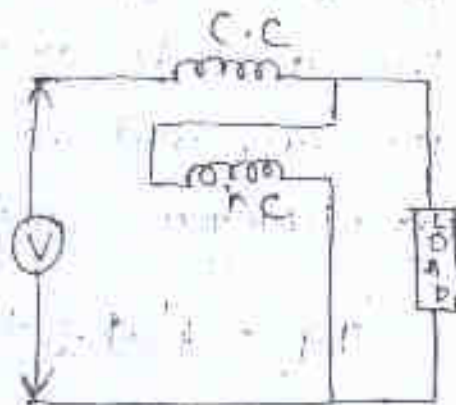
$$Z_p = j\omega (L - r^2 c) + R_p$$

$$Z_p = R_p$$

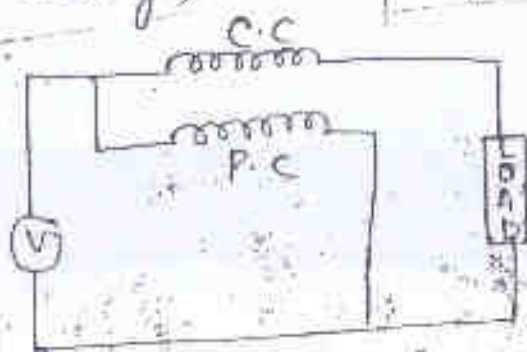
$$C = \frac{L}{r^2}$$



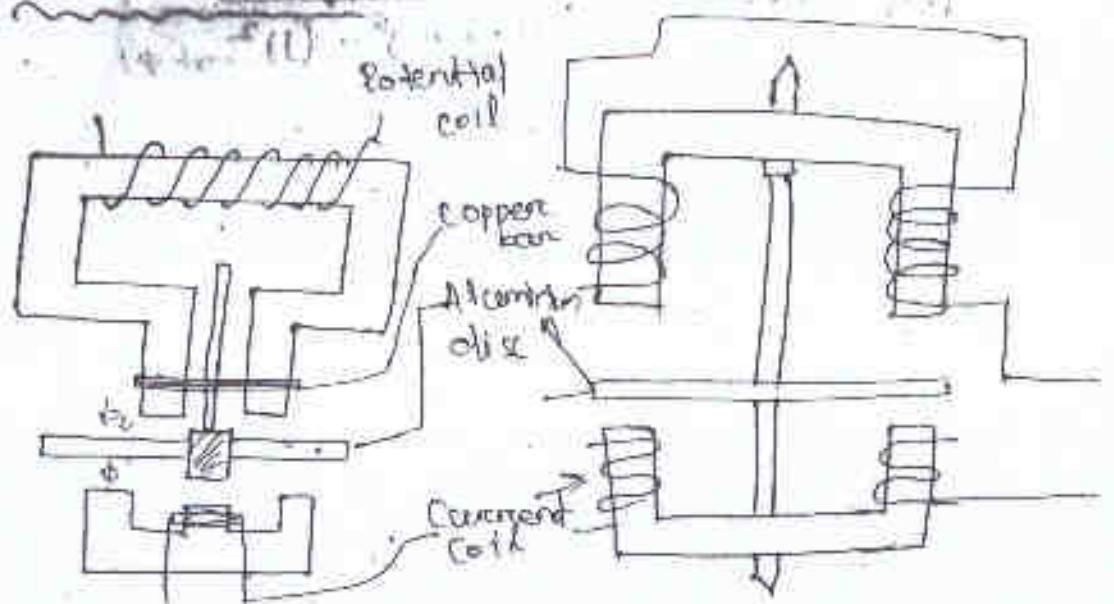
Power loading = Power load + Power
(low current rating)



Power load + Power P.C
(High current rating)

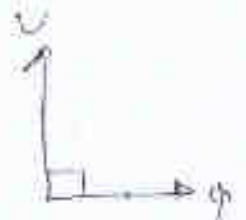


* Induction type wattmeter $V \cos \phi$ $\phi \approx 90^\circ$

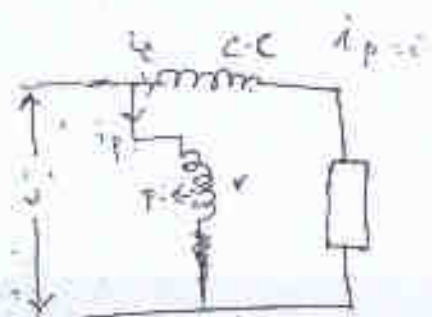


Q. The current coil of an electro-dynamic wattmeter in series with a 6Ω resistance is connected to a $24V$ DC supply. The pressure coil in series with an ideal diode is connected to 50 Hz sinusoidal supply of peak voltage $100V$. The pressure coil has a total resistance of 100Ω . Neglect the current coil resistance compute the wattmeter reading.

$$\begin{aligned}
 T_d &= k' \phi_{ce} \phi_{sh} \cos \alpha \cdot \sin \beta \\
 &= k' I \cdot V \cos \alpha \cdot \sin \beta \\
 &= k' P \sin \beta
 \end{aligned}$$



$T_d \propto P$

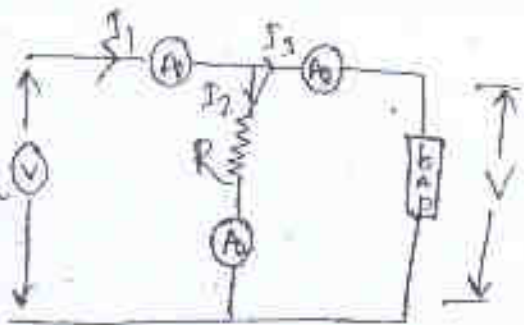


16/01/15

* Measurement of power in 1- ϕ circuit

① Using 3-Ammeter

Ammeter resistance negligible
voltmeter resistance very high



$$I_1^2 = I_2^2 + I_3^2 + 2I_2I_3 \cos \phi$$

$$V = I_2 R$$

$$\Rightarrow I_2 = \frac{V}{R}$$

$$I_1^2 = I_2^2 + I_3^2 + 2I_2 I_3 \cos \phi$$

$$\Rightarrow I_1^2 = I_2^2 + I_3^2 + 2 \frac{V}{R} I_3 \cos \phi$$

$$\Rightarrow \frac{P \times 2}{R} = I_1^2 - I_2^2 - I_3^2$$

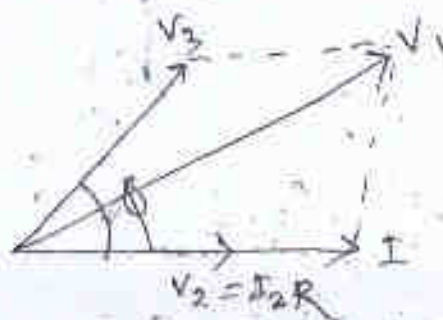
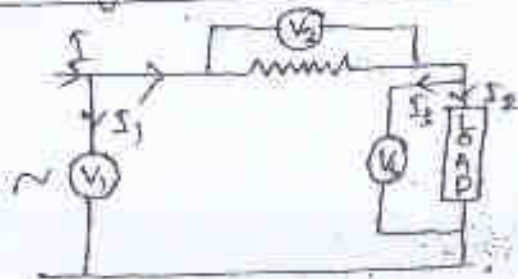
$$\Rightarrow P = \frac{(I_1^2 - I_2^2 - I_3^2) R}{2}$$

$$I_1^2 = I_2^2 + I_3^2 + 2I_2 I_3 \cos \phi$$

$$\Rightarrow \cos \phi = \frac{I_1^2 - I_2^2 - I_3^2}{2 I_2 I_3}$$

Inductive = current lags voltage lead
Resistive = in phase
Capacitive = current lead

② Using 3-Voltmeter



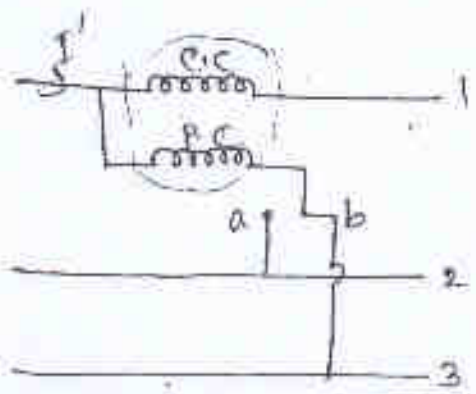
$$V_1^2 = V_2^2 + V_3^2 + 2V_2 V_3 \cos \phi$$

$$\Rightarrow V_1^2 = V_2^2 + V_3^2 + 2RI_2 V_3 \cos \phi$$

$$\Rightarrow V_1^2 = V_2^2 + V_3^2 + 2RP$$

$$\Rightarrow P = \frac{V_1^2 - V_2^2 - V_3^2}{2R}$$

* Using one watt-meter



b) w_1
a) w_2

$$E_{10} = E_{20} = E_{30} = E \text{ (say)}$$

Star

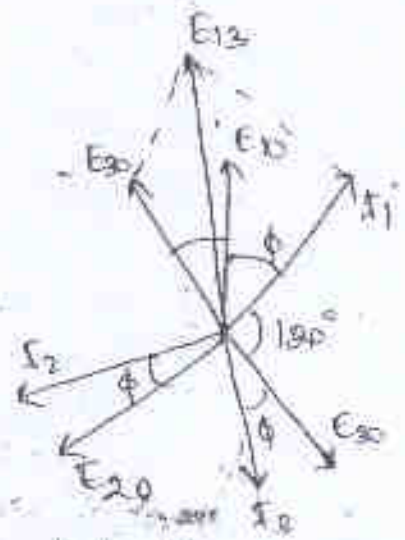
$$I_L = I_{ph}$$

$$V_L = \sqrt{3} V_{ph}$$

Delta

$$I_L = \sqrt{3} I_{ph}$$

$$V_L = V_{ph}$$

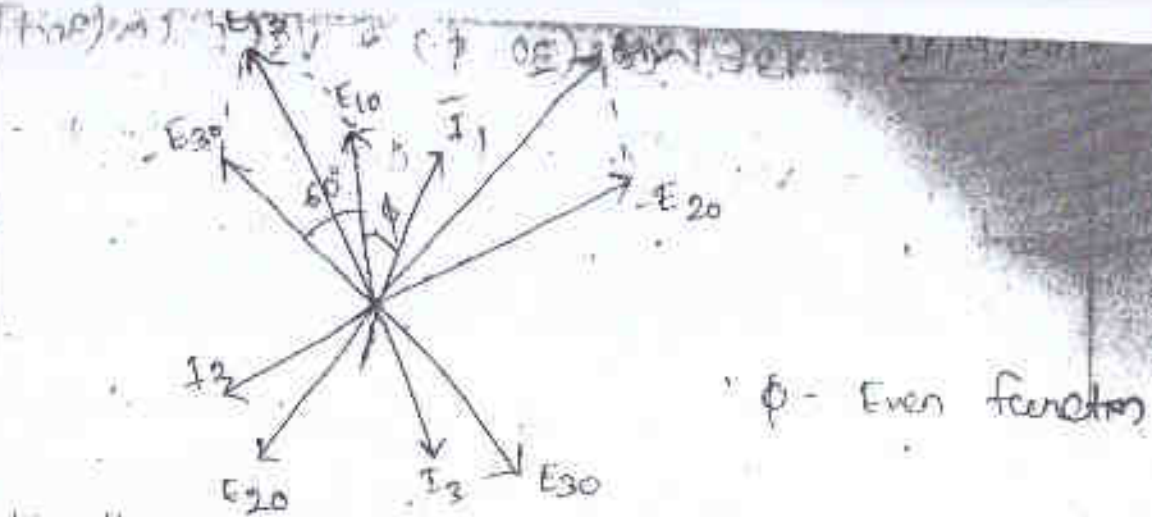


$$E_{10} + E_{30} = E_{10} - E_{30}$$

$$= E_{12}$$

$$E_{12} = \sqrt{3} E_{10}$$

$$E_{12} = \sqrt{3} E$$



Line voltage

$$E_{12} = \sqrt{E_{10}^2 + E_{20}^2 + 2E_1 E_2 \cos 60^\circ}$$

$$E_{12} = \sqrt{3} E \quad [E_{10} = E_{20} = E_{30} = E = \text{Phase voltage}]$$

$$W_1 = E_{12} I_1 \cos(\phi + 30^\circ)$$

$$W_1 = \sqrt{3} E I \cos(30^\circ + \phi) \quad [I_1 = I_2 = I_3 = I]$$

~~W2 =~~

$$E_{12} = \sqrt{3} E$$

$$W_2 = E_{12} I_1 \cos(\phi - 30^\circ)$$

$$= \sqrt{3} E I \cos(\phi - 30^\circ)$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$W_1 + W_2 = \sqrt{3} E I \cos(\phi - 30^\circ) + \sqrt{3} E I \cos(\phi + 30^\circ)$$

$$= 2 \sqrt{3} E I \cos \phi$$

$$= \sqrt{3} E I (\cos 30^\circ \cos \phi - \sin 30^\circ \sin \phi + \cos 30^\circ \sin \phi + \sin 30^\circ \cos \phi)$$

$$= \sqrt{3} E I (\cos 30^\circ \cos \phi)$$

$$= \sqrt{3} E I \cdot 2 \times \frac{\sqrt{3}}{2} \cdot \cos \phi$$

$$= 3 E I \cos \phi$$

$$\begin{aligned}
 \sqrt{3}EI \omega_1 &= \sqrt{3}EI \cos(30-\phi) - \sqrt{3}EI \cos(30+\phi) \\
 &= \sqrt{3}EI \left[\cos 30 \cdot \cos \phi + \sin 30 \cdot \sin \phi - \right. \\
 &\quad \left. (\cos 30 \cdot \overset{\cos}{\sin \phi} - \sin 30 \cdot \sin \phi) \right] \\
 &= \sqrt{3}EI \left[\cancel{\cos 30} \cdot \cancel{\cos \phi} + \sin 30 \cdot \sin \phi - \cos 30 \cdot \right. \\
 &\quad \left. \cos \phi + \cancel{\sin 30} \cdot \sin \phi \right] \\
 &= \sqrt{3}EI \cdot 2 \cdot \frac{1}{2} \sin \phi \\
 &= \sqrt{3}EI \sin \phi \quad \text{--- (2)}
 \end{aligned}$$

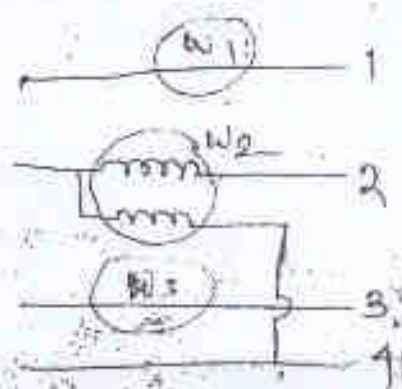
~~tan~~ Eq (2) by Eq (1) we get

$$\frac{\sqrt{3}EI \sin \phi}{3\sqrt{3}EI \cos \phi} = \frac{(\omega_2 - \omega_1)}{(\omega_2 + \omega_1)}$$

$$\Rightarrow \frac{\sin \phi}{\cos \phi} = \frac{\omega_2 - \omega_1 \times 3}{\omega_2 + \omega_1 \times \sqrt{3}}$$

$$\Rightarrow \tan \phi = \frac{\sqrt{3}(\omega_2 - \omega_1)}{\omega_2 + \omega_1}$$

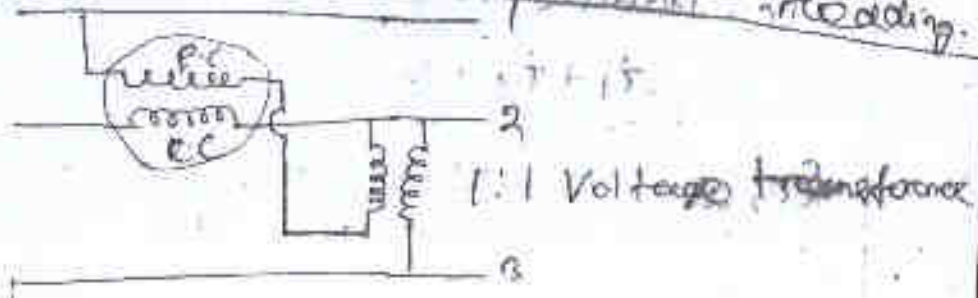
① 3- ϕ Ferris wire system (BARLOW METHOD)



$\omega_1, \omega_2, \omega_3$

$$\Rightarrow \omega_1 + \omega_2 + \omega_3 = \omega$$

By using voltage transformer measure wattmeter reading.

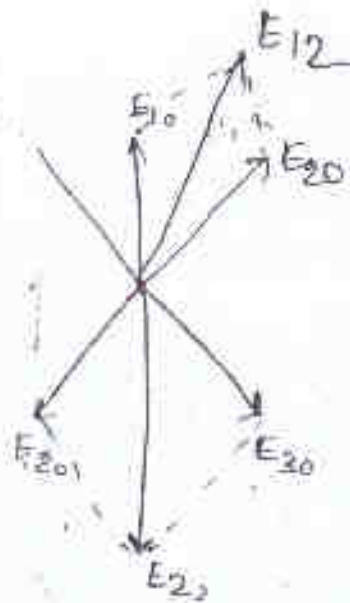


$$E_{12} = E_{23} = E_{31} = V_L$$

$$E_{10} = E_{20} = E_{30} = E = V_{Ph}$$

$$E_{12} = \sqrt{3} E \quad \sqrt{3} (E_{12})$$

$$E_{23} = \sqrt{3} E \quad \sqrt{3} (V_{Ph}) \\ = 3E$$



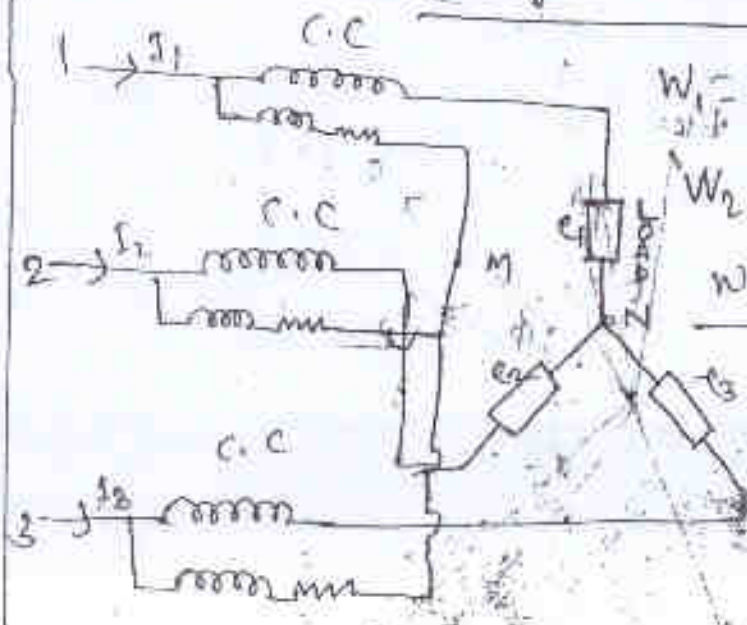
Resultant of E_{12} and E_{23}

$$\therefore E_p = \sqrt{3} (E_{12}) \\ = \sqrt{3} (\sqrt{3} E) \\ = 3E$$

Wattmeter reading $\times E_p \times I \cos \phi$

$$= 3E I \cos \phi$$

For balance (Using 3 wattmeter method)



$$W_1 = (e_1 \pm V) i_1$$

$$W_2 = (e_2 \pm V) i_2$$

$$W_3 = (e_3 \pm V) i_3$$

$$= e_1 i_1 + e_2 i_2 + e_3 i_3 \pm V (i_1 + i_2 + i_3)$$

$$\left[\begin{array}{l} + \text{ if } V_m \angle V_L \\ - \text{ if } V_m \angle V_L \end{array} \right]$$

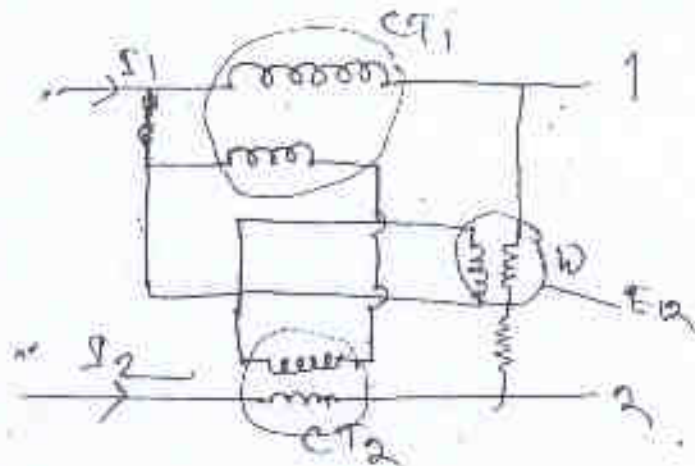
Problem: 1. In a three phase system

$$i_1 + i_2 + i_3 = 0$$

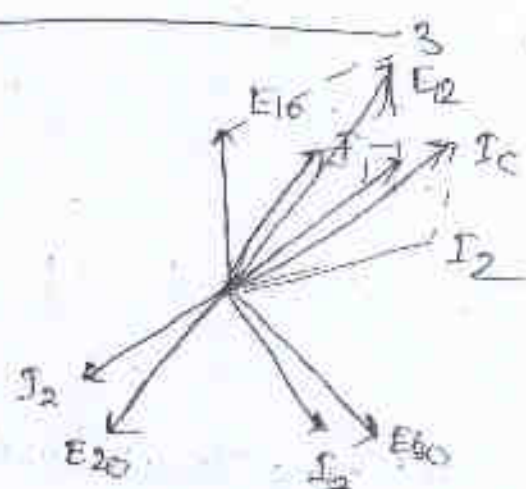
Sol: $\omega_1 = \omega, \rightarrow \omega_2 + \omega_3 = \text{Ang of } (e_1 i_1 + e_2 i_2 + e_3 i_3)$

Date 21/01/15

BARLOW METHOD - 2 Using current transformer



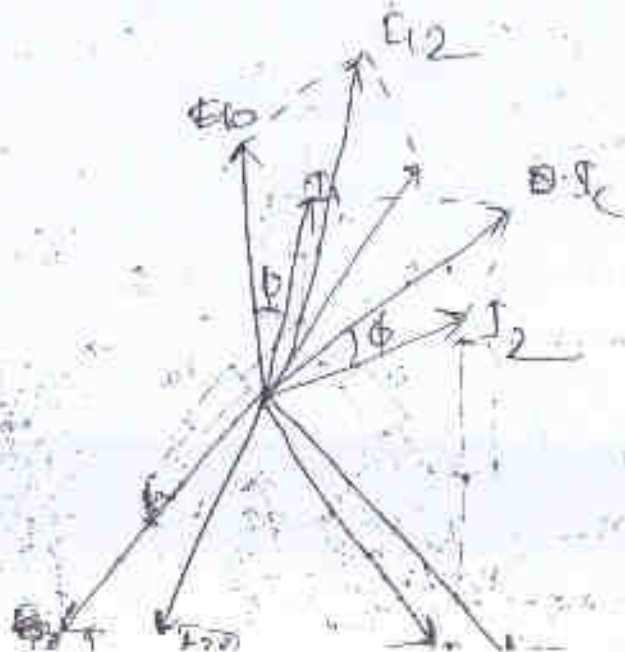
current transformer



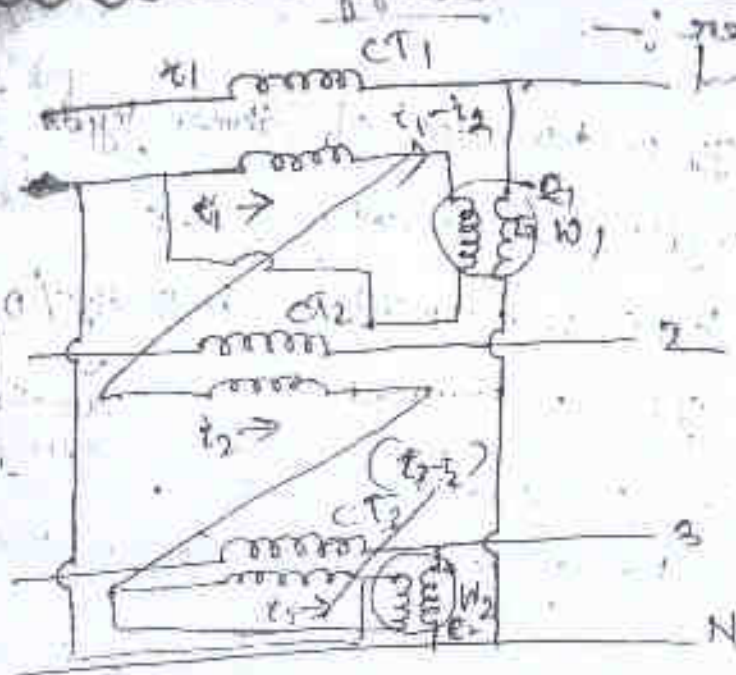
$$W = E_{12} I_c \cos \phi$$

$$= \sqrt{3} E V \cos \phi$$

$$= 3 E I \cos \phi$$



Two wattmeter method



$$e_1 (i_1 - i_2) + e_3 (i_3 - i_2)$$

$$= e_1 i_1 + e_3 i_3 - i_2 (e_1 + e_3)$$

$$e_1 + e_2 + e_3 = 0 \quad (\because \text{Balance condition})$$

$$\Rightarrow -i_2 = e_1 + e_3$$

$$= e_1 i_1 + e_2 i_2 + e_3 i_3$$

* Why calibration is necessary in measuring instruments?

Ans: Calibration is necessary in measuring instrument to get the static performance characteristics common from one another.

* Multiplying factor: Multiplying factor

in a wattmeter is defined as the ratio between the I/P power and load power.

$$m = \text{multiplying factor} = \frac{P_i}{P_o} = \frac{P_{\text{input}}}{P_{\text{output}}}$$

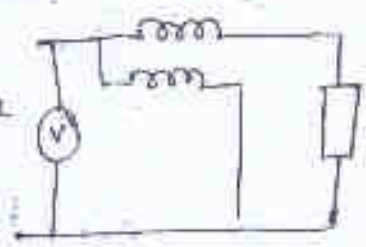
Energy meter

Energy meter are of three types

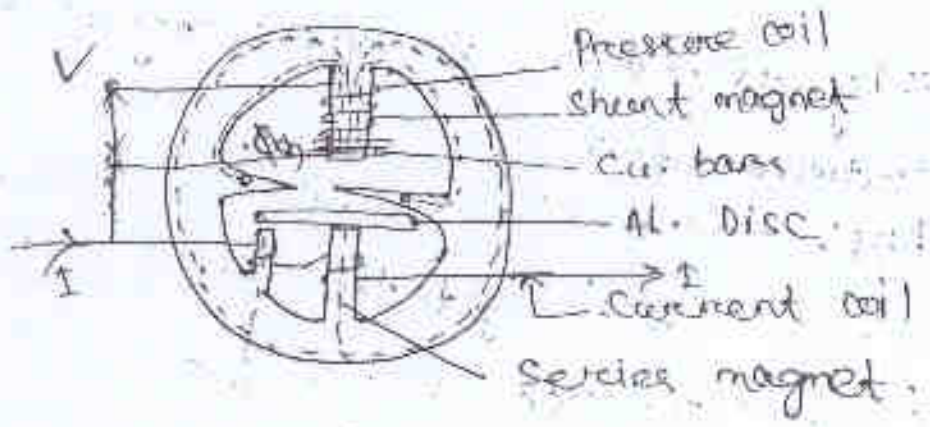
- (i) Electrolytic instruments - D.C
- (ii) Motor type instruments - both A.C/D.C
 - ↓
 - Induction motor type
 - ├── Commutator motor type
 - └── Mercury motor type
- (iii) clock type instruments -
 - ↓
 - Construction - complicated
 - Rarely used

* Induction motor type :- (1- ϕ)

- (i) Driving System \rightarrow Electromagnet
- (ii) Moving System \rightarrow Aluminium disc
- (iii) Braking System \rightarrow Permanent magnet
- (iv) Registering System \rightarrow Reduction gear



(i) Driving System :-



Φ

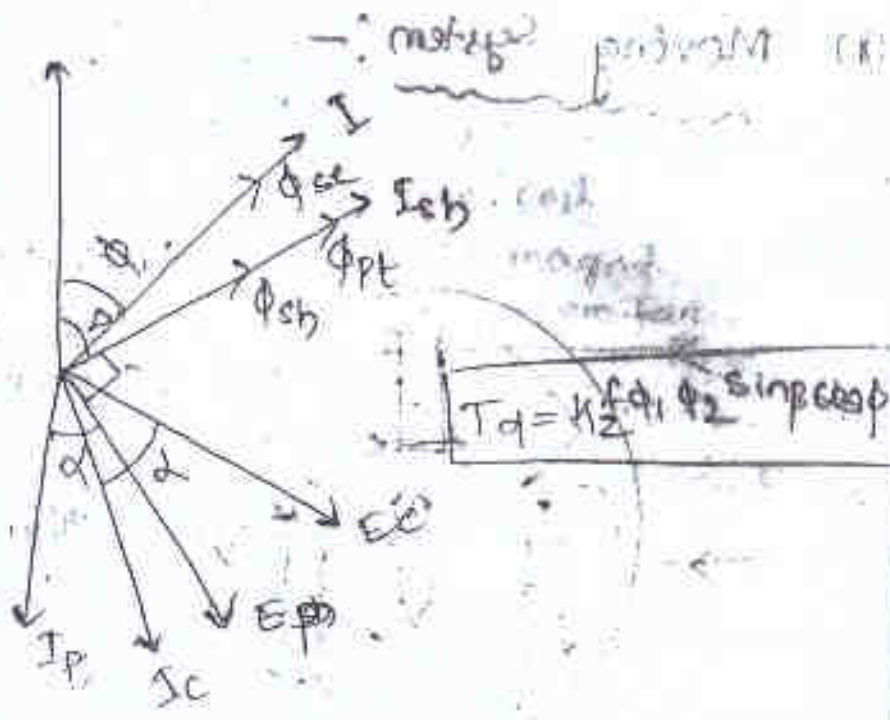
Torque = Eddy current \times flux

$$\phi_{PT} - \phi_g$$

$$\phi_{sh}$$

$$\phi_g \lll 1$$

$$\phi_{sh} \ggg 1$$



$$T_d = k_2 f \phi_1 \phi_2 \sin \beta \cos \alpha$$

$$T_d = k_2 f \phi_1 \phi_2 \sin \beta \cos \alpha$$

$$= k_1 \phi_{sh} \phi_{se} \sin(\Delta - \phi) \cdot \cos \alpha$$

$$= k_1 \phi_{sh} \phi_{se} \sin(\Delta - \phi) \quad (\because \cos \alpha \text{ is constant})$$

$$= k_1 V I \sin(\Delta - \phi) \quad (\because \phi_{sh} \propto V$$

$$\phi_{se} \propto I)$$

$$\Delta = 90^\circ$$

$$= k_1 V I \cos \phi$$

* Overload - The rated burden is the volt-Amp. loading which is permissible to the transformer exceeding the limits for the particular class of accuracy.

Total secondary burden = $\frac{(\text{Secondary vol. induced volt.})^2}{\text{Impedance of the secondary wdg. ckt including impedance of secondary wdg.}}$

$$T_d = k \frac{r}{Z} \phi_1 \phi_2 \sin \beta \cos \alpha$$

$$= k' \phi_{sh} \phi_{se} \sin(\Delta - \phi) \cdot \cos \alpha$$

$$= k' VI \sin(\Delta - \phi) \quad \left(\begin{array}{l} \because \phi_{sh} \propto V \\ \phi_{se} \propto I \end{array} \right)$$

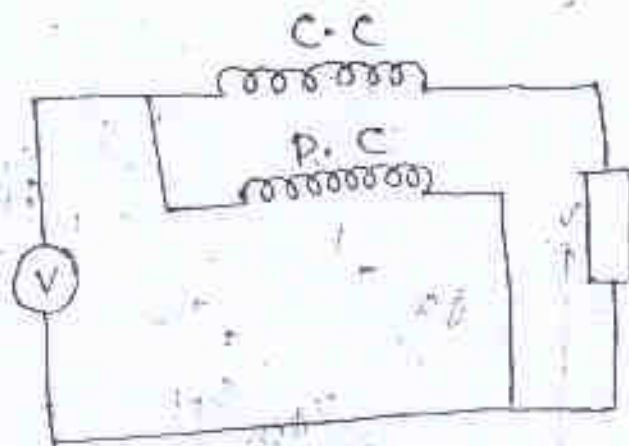
Let: $\Delta = 90^\circ$

$$T_d = k' VI \cos \phi$$

(Pressure coil pure inductive)

$$T_B = T_D$$

$$\Rightarrow VI \cos \phi dN = \frac{\text{No. of Revolution}}{dt \text{ (minut)}}$$



$$\phi_{CC} \propto I_{CC}$$

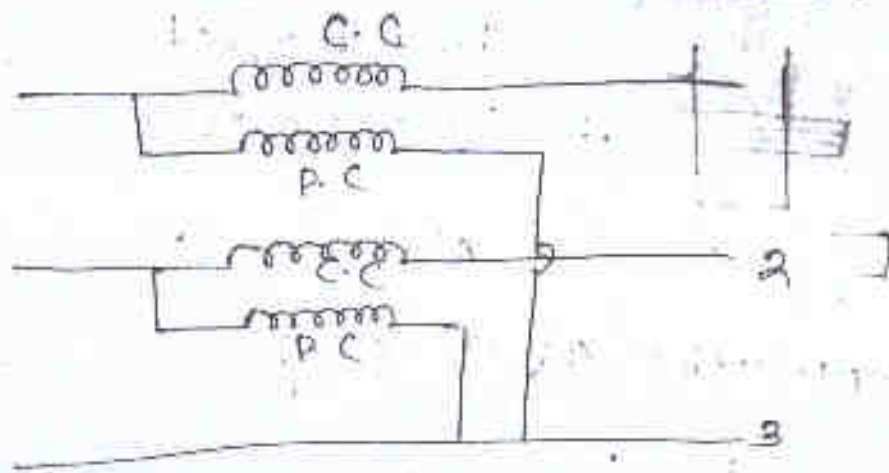
$$\phi_{PC} \propto V$$

$\rightarrow dt = \text{no. of revolution}$

$$\Rightarrow \int (VI \cos \phi) dt = \text{Total revolution}$$

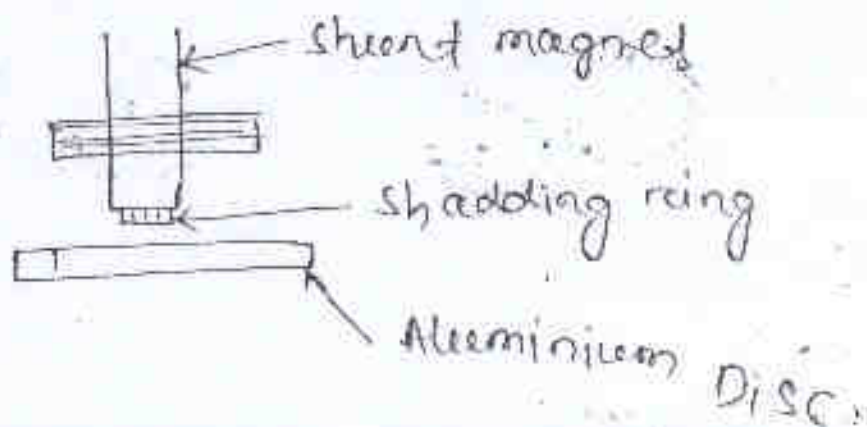
$$\Rightarrow \text{Total kWh} = \text{Total revolution}$$

Polyphase (3 ϕ) Energy meter

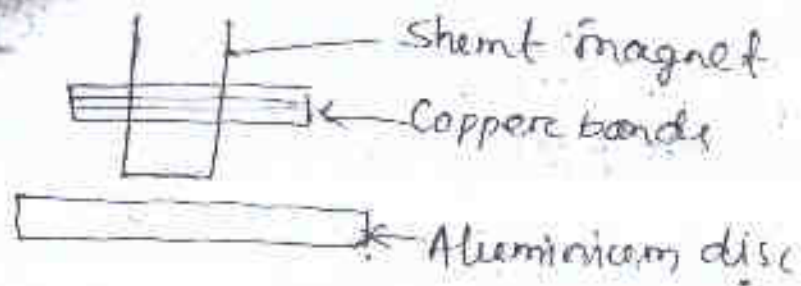


* Errors :-

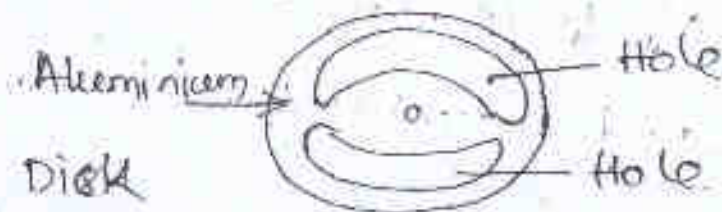
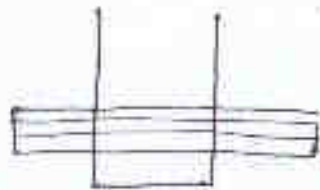
- ① Compensation of frictional errors
- ② Phase error and inductive load compensation.
- ③ Compensation of creeping.
- ④ Temperature compensation.
- ⑤ Compensation of frictional errors



2 - phase creep



3 - Compensation of creeping :-



④ Temperature Compensation :-

* Sheaf temperature magnet...

* The wheel gear is distance



Inductive \rightarrow Current lags voltage

Capacitive \rightarrow Current leads voltage

Resistor \rightarrow In phase

$V I \cos \phi \rightarrow$ True Power

$V I \sin \phi \rightarrow$ Reactive Power

$V I \cos \phi \rightarrow$ Active Power

Power factor D.C or A.C

only A.C \rightarrow Power factor

watt - Uniform

Shunt - P.C parallel with load

Series - C.C series with load

thick - High current

thin - low current

Testing of Energy Meter:-

Date 28/01/2015

Standard meter: n_1 rev - k_1 rev for kWh
 Substandard meter: n_2 rev - k_2 rev for kWh

Constant
 Error: $\frac{\frac{n_2}{k_2} - \frac{n_1}{k_1}}{\frac{n_2}{k_2}} = \frac{n_2 k_1 - n_1 k_2}{n_2 k_1} \times \frac{k_1}{n_2}$

Error = $\frac{n_2 k_1 - n_1 k_2}{n_2 k_1}$

x The meter constant of a 230V, 10A watt hour meter is 1800 rev for kWh. The meter is tested at half load at rated voltage and with ^{unit} power factor. The meter is found to be make 180 revolutions in 138 sec. Determine the meters error at half load.

Solⁿ:- Voltage (V) = 230V
 Current (I) = 10A
 time (t) = 138 sec

Again (V) = 230V
 I = 5A (for half load)
 cos ϕ = 1
 time = $\frac{138}{3600}$ h

$E = VI \cos \phi \cdot t$
 $= 230 \times 5 \times 1 \times \frac{138}{3600} = 44.08 \text{ Wh} \times 10^{-3} \text{ kWh}$
 (True value)
 $= 0.04 \text{ kWh}$

1800

1 kWh

50

$$\frac{1}{1500} \times 80 = 0.044 \text{ kWh}$$

$$= 44.44 \times 10^{-3} \text{ kWh (Originally)}$$

$$\text{Error} = \frac{41.41 - 44.08}{44.08} = 3.11\% \text{ fast}$$

$$\text{Error} = \frac{41.41 - 44.08}{44.08} = 0.0817\% \text{ fast}$$

* Frequency Meter :- (A.C measurement)

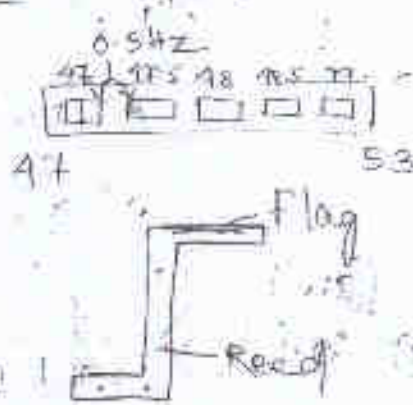
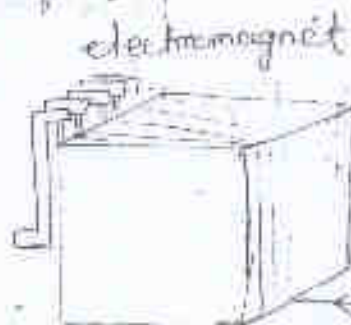
① Mechanical reed type (mechanical resonance type)

② Deflection type (Electrical resonance type)

③ Mechanical resonance type -

Natural frequency
= 2x f_{sup}

width = 4mm
thickness = 0.5mm



Advantages

- ① Don't notice when apply small voltage
- ② Don't measure accurate frequency (ie. when 50 Hz)

disadvantages

- ① It don't need any shape or size

Q. A standard energy meter makes 500 rev. per kWh.
It is found on testing at making 40 rev. in 58.1 sec

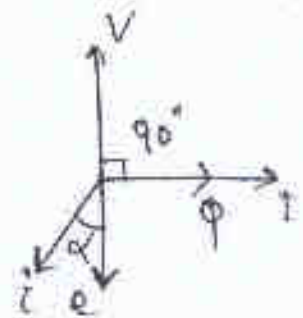
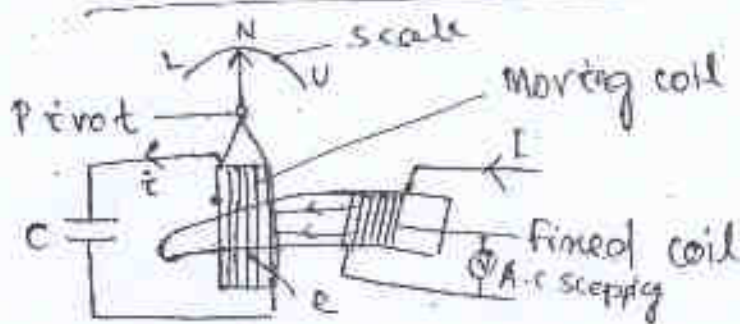
at a kw full load. Find out the percentage error.

Q. The meter which const. is 750 rev per kWh makes 15 rev. in 20 sec. Determine the load in kw.

Date 29/01/15

Electrical Resonance type:-

① Ferrodynamic type instrument

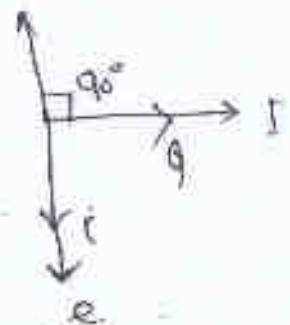


ϕ (flux) is in phase with I (current)
 e lag ϕ by 90° .

Case 1

$$① \quad 2\pi fL > \frac{1}{2\pi fC}$$

$$X_L > X_C$$



$$T_d \text{ (deflecting torque)} = i_1 i_2 \cos \phi$$

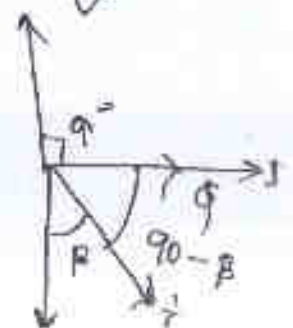
$$= I i \cos (90^\circ + \alpha)$$

$$= I i \cos 90^\circ \quad (\alpha = 0)$$

$$= 0$$

Case - 2

$$2\pi fL < \frac{1}{2\pi fC}$$



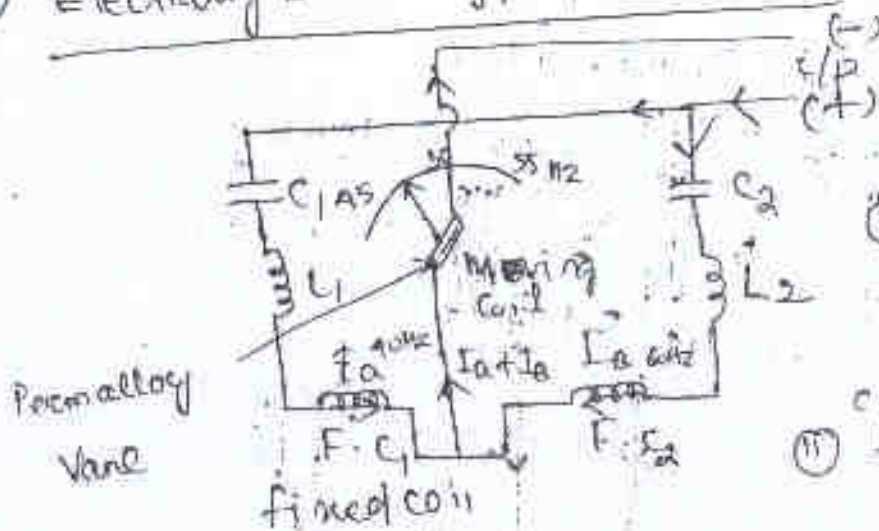
$$T_d = I_1 I_2 \cos(\theta_0 - \beta)$$

$$= I_1 I_2 \cos 90 \quad (\beta = 0)$$

$$T_d = 0$$

frequency

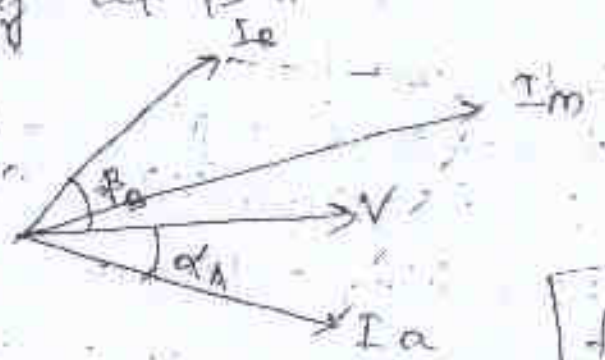
① Electrodynamometer type instrument



- Inductive
- capacitive
- ① $f_c = 40 \text{ Hz} < 50 \text{ Hz}$
- $2\pi fL > \frac{1}{2\pi fC}$
- ② $f_c = 60 \text{ Hz} > 50 \text{ Hz}$
- $2\pi fL < \frac{1}{2\pi fC}$

$$T_d = r_1 r_2 \frac{F^2}{Z^2} \cos(\phi)$$

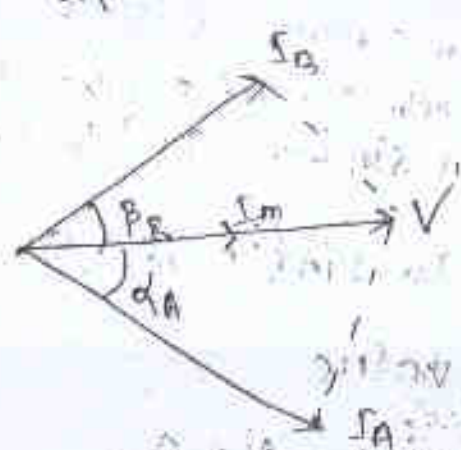
Supply at 45°



$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

resonant frequency

Supply at 50 Hz



resonant frequency

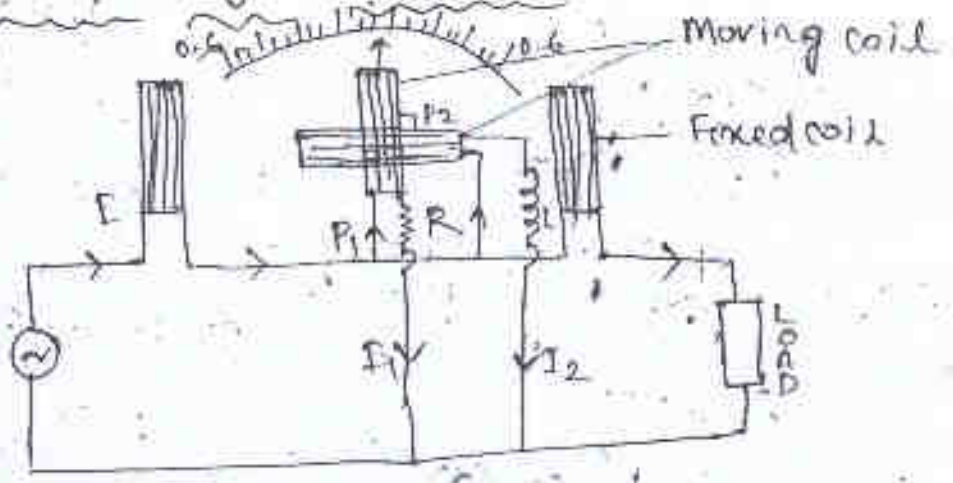
* Power Factor Meter

- (1) Single phase
- (2) Three phase

Instruments

- (1) Dynamometer type
- (2) Moving iron type

* Dynamometer type instruments



$P_1 =$ Moving coil 1
 $P_2 =$ Moving coil 2

let,
 (1) $i_1 \propto V_m \sin \omega t$
 (2) $i_2 \propto V_m \sin (\omega t - 90^\circ)$
 Moving coil '1' & fixed coil (Z_1)
 Moving coil '2' & fixed coil (Z_2)

$T_1 \propto i_1 \propto V_m \sin \omega t \quad i \propto I_m \sin (\omega t - \phi)$

$\propto \{ V_m \sin \omega t \cdot I_m \sin (\omega t - \phi) \} \sin \theta$

$T_2 \propto \{ V_m \sin (\omega t - 90^\circ) \cdot I_m \sin (\omega t - \phi) \} \cos \theta$

$\propto V_m I_m \sin (\omega t - 90^\circ) \sin (\omega t - \phi) \cos \theta$

Taking average value

$T_1 \propto V_m I_m \cos \phi \cdot \sin \theta$

$T_2 \propto V_m I_m \sin \phi \cdot \cos \theta$

In equilibrium condition $\theta = \phi$

$$V_m I_m \cos \phi \cdot \sin \theta = V_m I_m \sin \phi \cdot \cos \theta$$

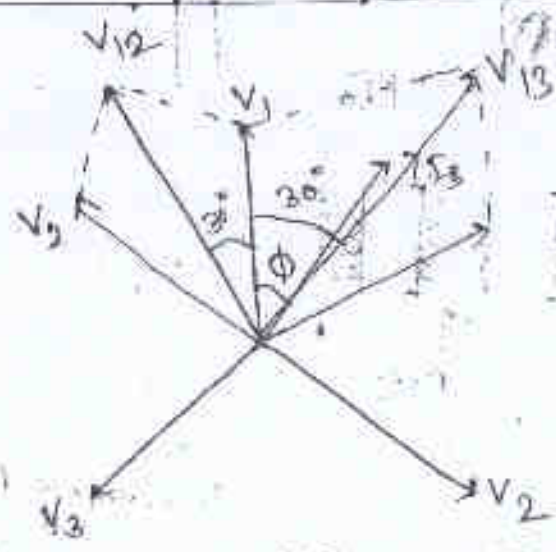
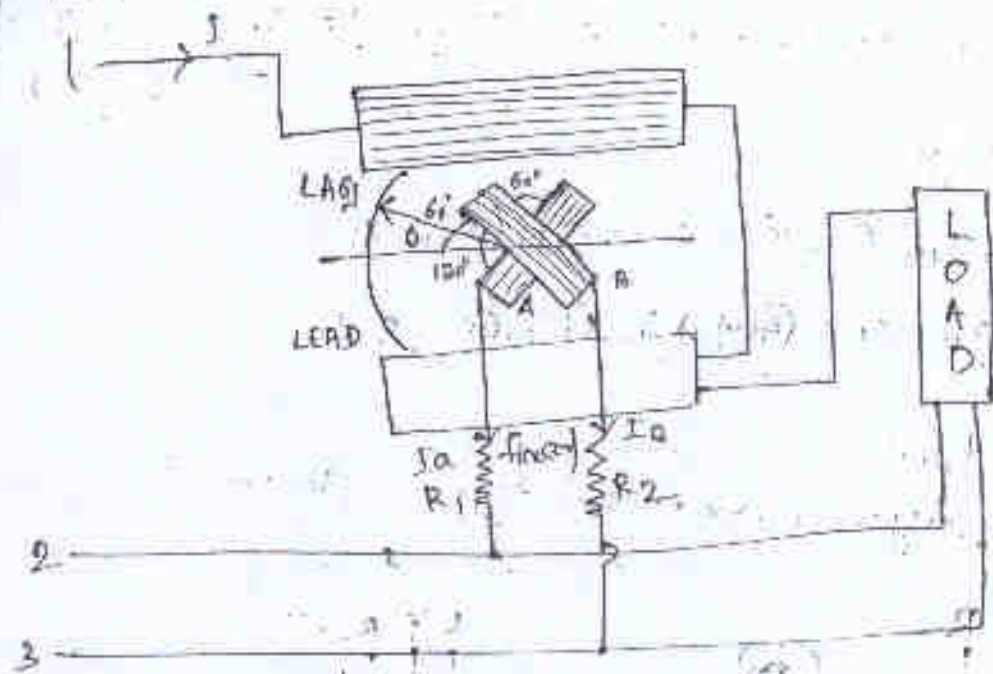
$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{\sin \phi}{\cos \phi}$$

$$\Rightarrow \tan \theta = \tan \phi$$

$$\Rightarrow \theta = \phi$$

Date: 31/01/2015

3 ϕ Power factor meter



$I_A \propto V_m \sin(\omega t + \theta)$
 $I_B \propto V_m \sin(\omega t + \theta + 120^\circ)$

$\tau_A \propto I_m \sin(\omega t - \phi)$ (Instantaneous value)
 $\tau_B \propto V_m \sin(\omega t - 30^\circ) I_m \sin(\omega t - \phi) \sin(120 - \theta)$
 $\tau_C \propto V_m I_m \sin(\omega t + 90^\circ) \sin(\omega t - \phi) \sin(120 - \theta)$
 $\tau_B \propto V_m \sin(\omega t - 30^\circ) \sin(\omega t - \phi) \sin(120 - \theta)$
 $\Rightarrow \tau_B \propto V_m I_m \sin(\omega t - 30^\circ) \sin(\omega t - \phi) \sin(60 - \theta)$

Taking avg value

$\tau_A \propto V_m I_m \cos(30 + \phi) \cdot \sin(120 - \theta)$
 $\tau_B \propto V_m I_m \cos(90 - \phi) \cdot \sin(60 - \theta)$

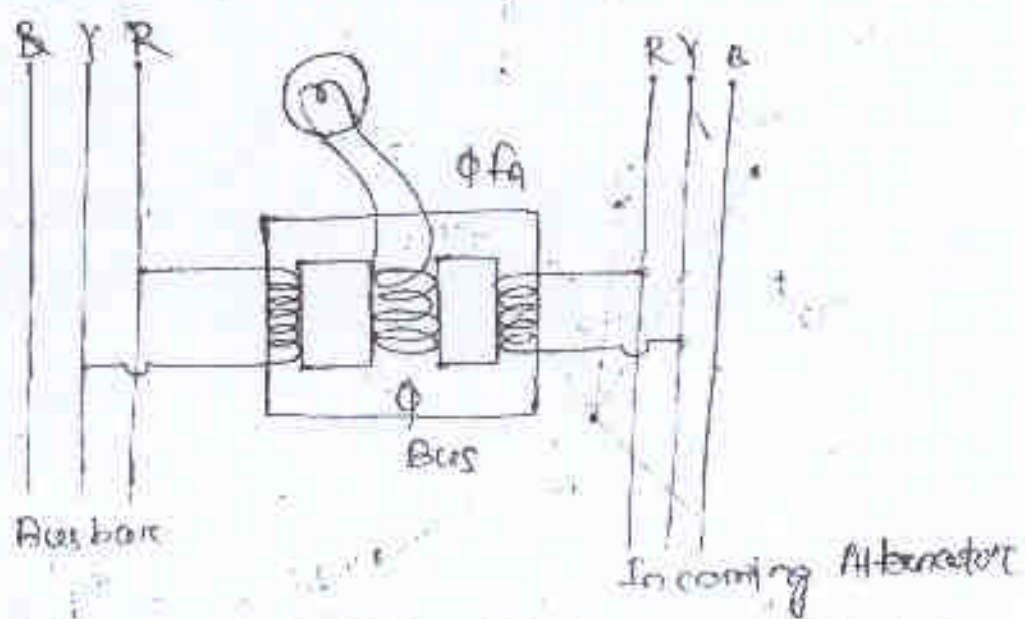
At equilibrium position

$\tau_A = \tau_B \Rightarrow \cos(30 + \phi) \cdot \sin(120 - \theta) = \cos(90 - \phi) \sin(60 - \theta)$

Phase change

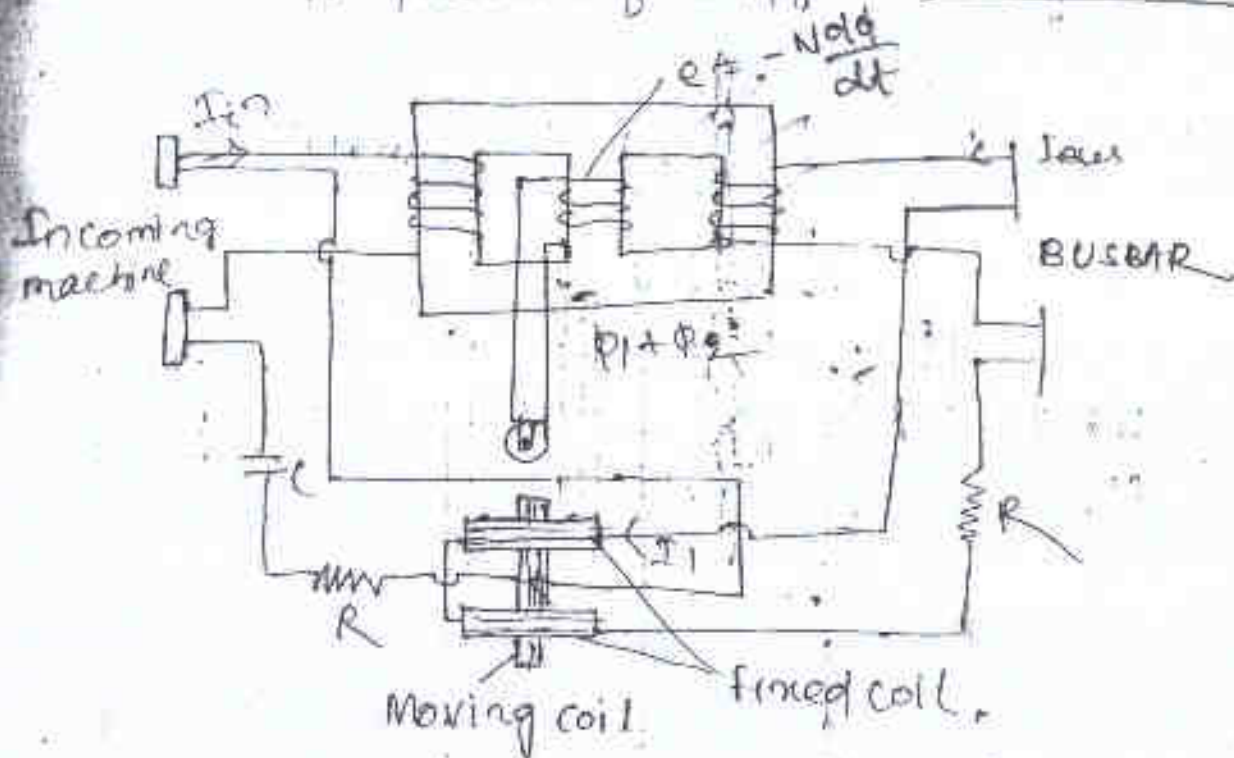
$\tau_A \propto V_m I_m \cos(30 - \phi) \cdot \sin(120 - \theta)$
 $\tau_B \propto V_m I_m \cos(30 + \phi) \sin(60 - \theta)$

- ① M.I type
- ② Electrodynamometer wattmeter type instrument



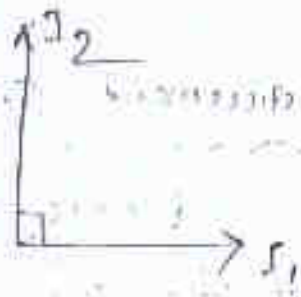
If there is no phase difference $\phi = 0$

Resultant flux = $\Phi_A + \Phi_B$



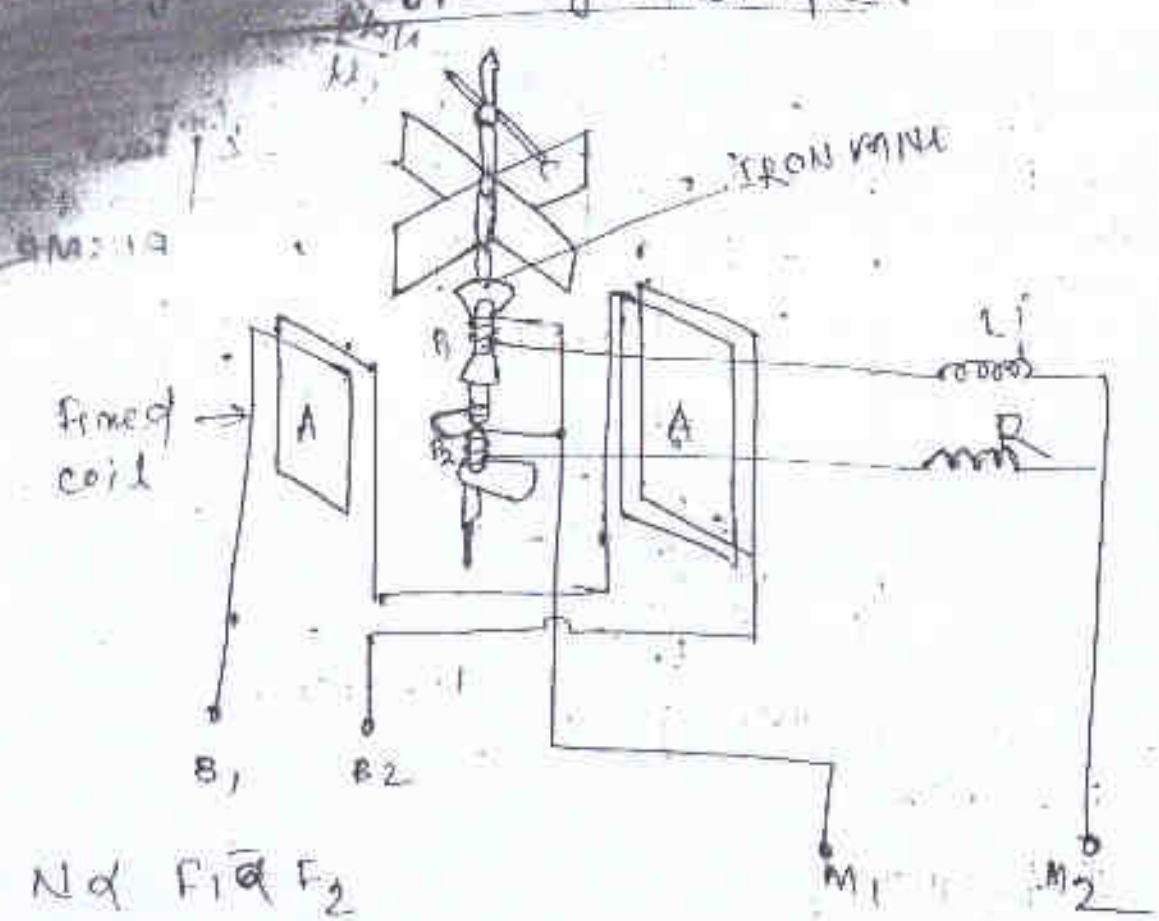
- ① Voltage
- ② Frequency
- ③ Phase sequence

$$\begin{aligned} \text{Torque} &= I_1 I_2 \cos \phi \\ &= I_1 I_2 \cos 90^\circ \\ &= 0 \quad (\because \cos 90^\circ = 0) \end{aligned}$$



In phase $\rightarrow \phi_1 + \phi_2 \rightarrow$ glow
 out of phase $\rightarrow -\phi_1 + \phi_2 \rightarrow$ not glow

2101 Series Iron type synchroscope :-



Dt 03-02-2015

* Instrument Transformer :-

Power and frequency are constant in a transformer.

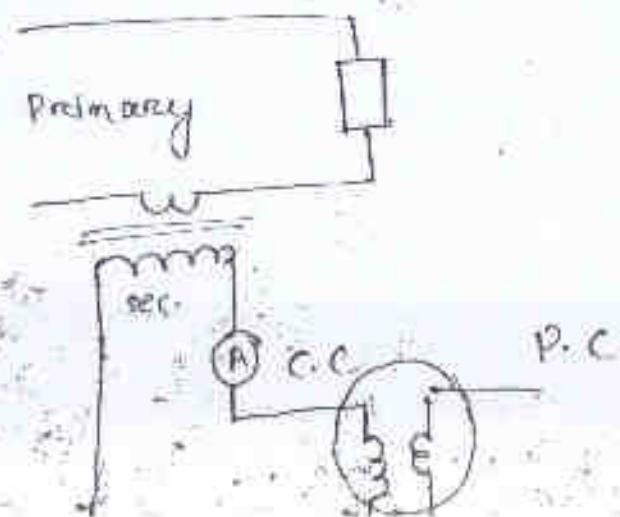
Step Up \rightarrow No. of turns high

$$N_1 < N_2$$

$$V_1 < V_2$$

$$I_1 > I_2$$

* Current Transformer :-



$$n = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$\Rightarrow I_1 = n I_2$$

Phasor diagram of core type transformer

① Core :- (a) Low reluctance

$$\text{Reluctance } (\mathcal{R}) = \frac{l}{\mu_0 \mu_r A}$$

(b) Low loss

→ Area of cross-section ↑

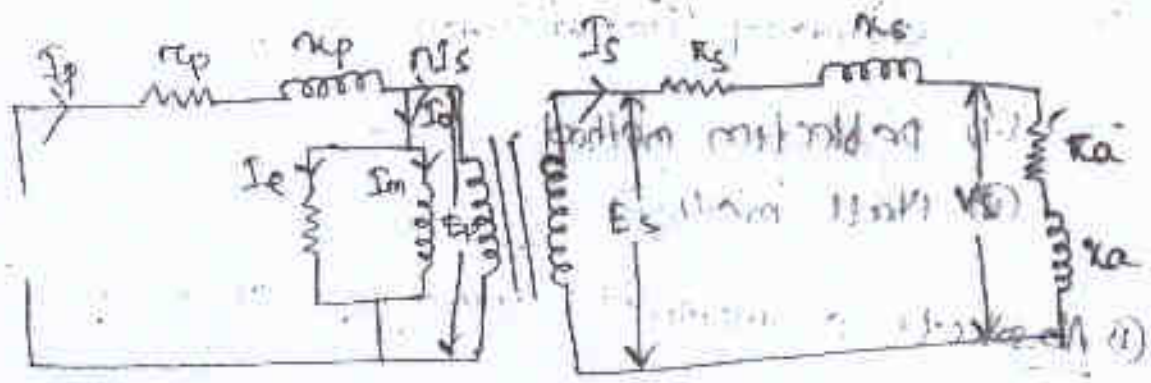
→ Flux density is less

→ Shorter magnetic paths

→ Hot rolled silicon steel

→ CRGO → (Cold rolled grain oriented silicon steel)

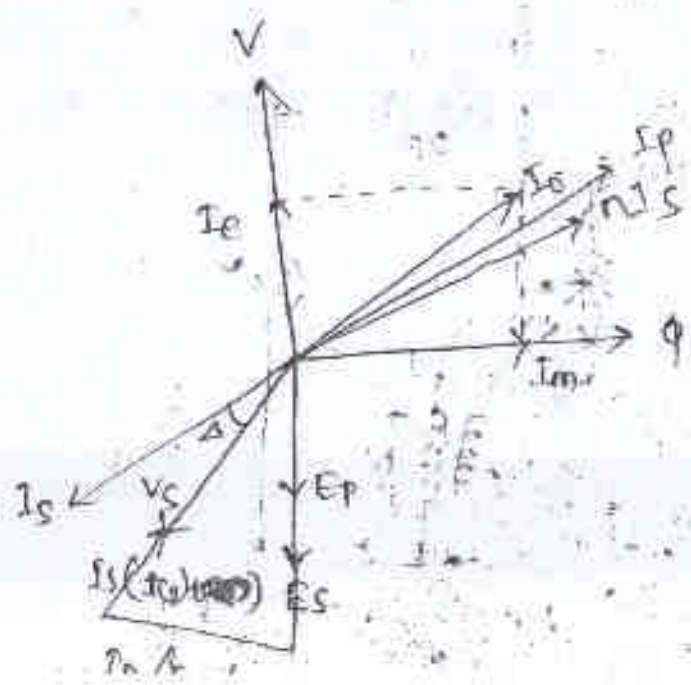
→ Ni-Iron alloy



② Winding :- (i) sandal type winding
 (ii) Bar type winding

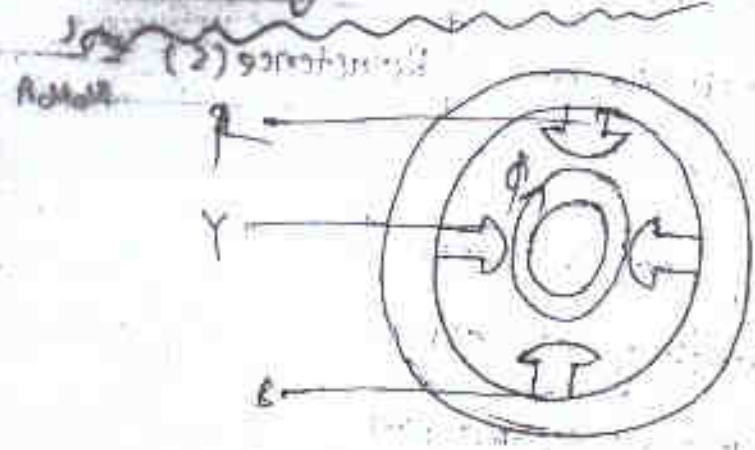


*



Phase sequence indicator

Date 09/02/2019



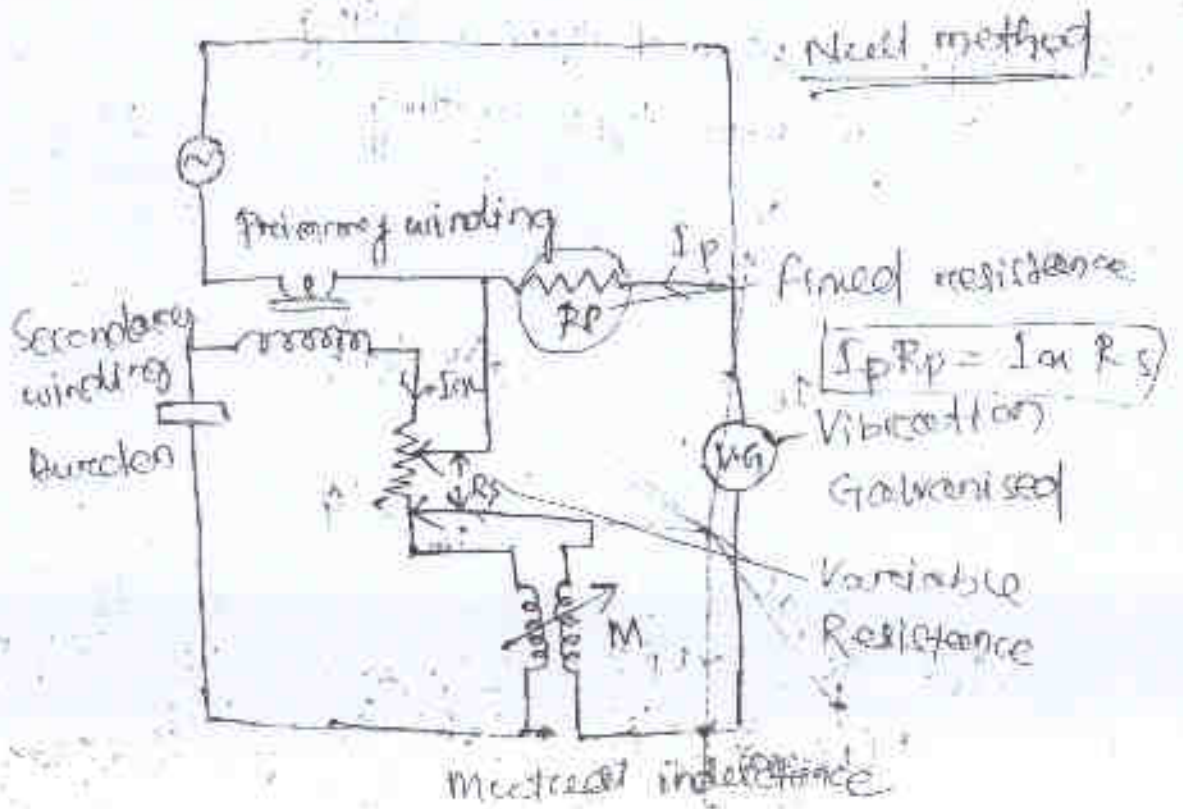
Date 09/02/2015

Testing of instrument Transformer

- ① Absolute Method
 - ② Comparison method
- standard Transformer

- (i) Deflection method
- (ii) Null method

① Absolute or mutual inductance method



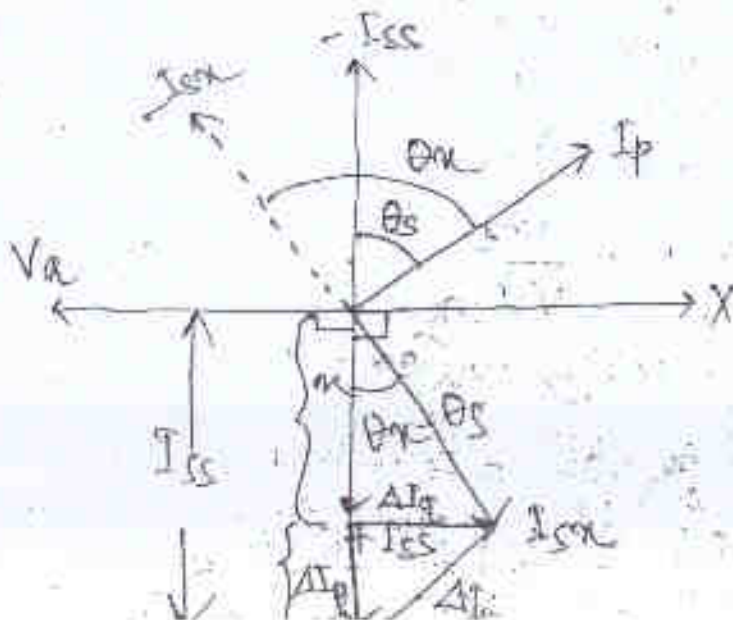
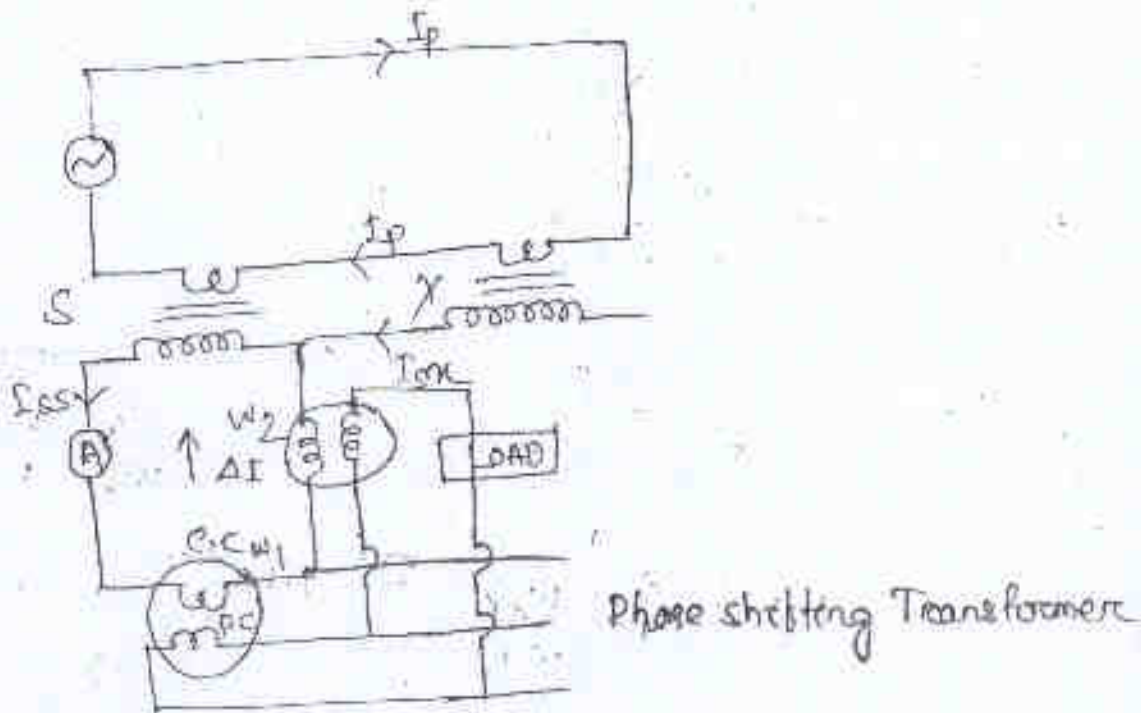
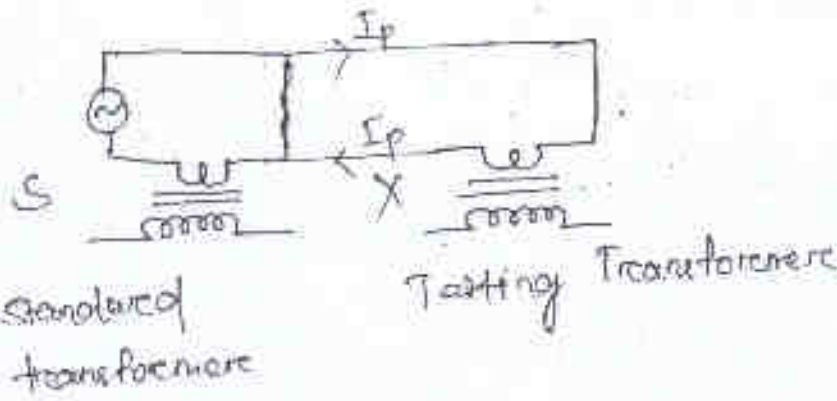
$$I_p R_p = I_n R_s$$

$$\Rightarrow R_s = \frac{I_p R_p}{I_n}$$

$$S = \omega p_2 \omega_1 I_p = I_p^2 \omega$$

Date 10/02/2015

Silsbee's Method :- Comparison Method of ω



$$I_n (\theta_n - \theta_m) I_n$$

$$\Delta I_p = I_n \sin \theta_n$$

$$W_{10} = V_p I_{sc} \cos \theta_0 = 0$$

$W_{10} = V_q \Delta I$ component, in phase with V_q

$$= V_q I_{sc} \sin(\theta_x - \theta_s) \quad \text{--- (i)}$$

$$\Rightarrow \sin(\theta_x - \theta_s) = \frac{W_{2q}}{V_q I_{sc}}$$

$$W_{1P} = V_p I_{ss} \cos 0^\circ = V_p I_{ss}$$

$W_{2P} = V_p \Delta I_P$ component in phase with V_p

$$= V_p [I_{ss} - I_{sc} \cos(\theta_x - \theta_s)]$$

$$= V_p I_{ss} - V_p I_{sc} \cos(\theta_x - \theta_s)$$

$$\Rightarrow W_p I_{sc} \cos(\theta_x - \theta_s) = W_{1P} - W_{2P}$$

$$W_{2P} = W_{1P} - V_p I_{sc}$$

$$\Rightarrow I_{sc} = \frac{W_{1P} - W_{2P}}{V_p}$$

$$R_x = \frac{I_P}{I_{sc}} \theta, \quad R_s = \frac{I_P}{I_{ss}}$$

$$\Rightarrow \frac{R_x}{R_s} = \frac{I_P}{I_{sc}} \times \frac{I_{ss}}{I_P} = \frac{V_p I_{ss}}{V_p I_{sc}} = \frac{W_{1P}/W_p}{\frac{W_{1P}}{W_p} \frac{W_{2P}}{W_{1P}}}$$

$$= \frac{1}{1 - \frac{W_{2P}}{W_{1P}}} \approx 1 + \frac{W_{2P}}{W_{1P}}$$

$$\Rightarrow R_x = R_s \left(1 + \frac{W_{2P}}{W_{1P}} \right)$$

$$W_{2q} = V_q I_{sc} \sin(\theta_x - \theta_s)$$

$$\Rightarrow \sin(\theta_x - \theta_s) = \frac{W_{2q}}{V_q I_{sc}} \quad \text{--- (1)}$$

$$W_{2P} = W_{1P} - V_p I_{sc} \cos(\theta_x - \theta_s)$$

$$\Rightarrow V_p I_{sc} \cos(\theta_x - \theta_s) = W_{1P} - W_{2P}$$

$$\Rightarrow \cos(\theta_x - \theta_s) = \frac{W_{1P} - W_{2P}}{V_p I_{sc}} \quad \text{--- (2)}$$

Eq (1) & (11) Divided

$$\sin(\theta_m - \theta_s) = \frac{\omega_{2q}}{V I_m}$$

$$\cos(\theta_m - \theta_s) = \frac{\omega_{1p} - \omega_{2p}}{V I_m}$$

$\frac{Eq(1)}{Eq(11)}$ we get

$$\tan(\theta_m - \theta_s) = \frac{\omega_{2q}}{\omega_{1p} - \omega_{2p}}$$

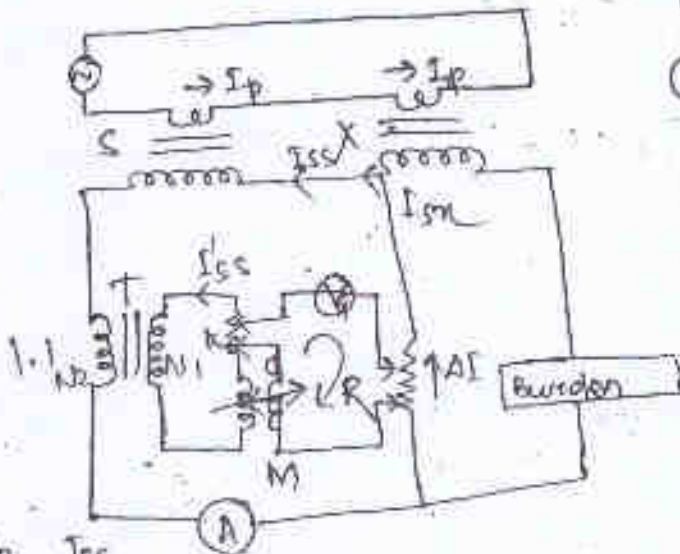
$$\Rightarrow \theta_m = \frac{\omega_{2q}}{\omega_{1p} - \omega_{2p}} + \theta_s$$

Date 12/02/2015

Airond's Method :-

① Comparison

② Null method



$$\frac{N_2}{N_1} = \frac{I_{sc}}{I_{cs}}$$

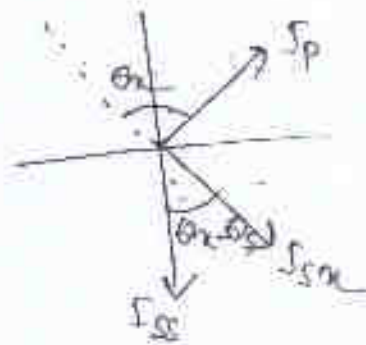
$$-I_{sc} R + A I R + I_{sc} I_m M = 0$$

$$A I = I_{sc} - I_m$$

$$\text{Actual Ratio } R_a = \frac{-I_p (\cos \theta_m + j \sin \theta_m)}{I_m}$$

$$R_a = \frac{-I_p (\cos \theta_m + j \sin \theta_m)}{I_m}$$

$$R_s = \frac{-I_p (\cos \theta_m + j \sin \theta_m)}{I_m}$$



$$R_x = \frac{I_p}{I_{sx}} (1 + j\theta_x) \quad R_s \text{ small} \quad \text{--- (I)}$$

$$R_s = \frac{-I_p}{I_{ss}} (1 + j\theta_s) \quad \text{--- (II)}$$

Now, dividing eqⁿ (I) & (II) we will get

$$\frac{R_x}{R_s} = \frac{-\frac{I_p}{I_{sx}} (1 + j\theta_x)}{-\frac{I_p}{I_{ss}} (1 + j\theta_s)}$$

$$\Rightarrow \frac{R_x}{R_s} = -\frac{I_p}{I_{sx}} (1 + j\theta_x) \times \frac{I_{ss}}{-I_p} (1 + j\theta_s)$$

$$\Rightarrow \frac{R_x}{R_s} = \frac{I_{ss}}{I_{sx}} (1 + j(\theta_x - \theta_s))$$

$$\Rightarrow I_{sx} = \left[\frac{R_s}{R_x} (1 + \theta_x - \theta_s) \right] I_{ss}$$

Now, putting the value of I_{sx} in $\Delta I = I_{ss} - I_{sx}$

$$\Rightarrow \Delta I = I_{ss} - I_{sx}$$

$$= I_{ss} - I_{ss} \left[\frac{R_s}{R_x} \{1 + j(\theta_x - \theta_s)\} \right]$$

$$= I_{ss} \left[1 - \frac{R_s}{R_x} \{1 + j(\theta_x - \theta_s)\} \right]$$

$$\Rightarrow I_{ss} \left[1 - \frac{R_s}{R_x} \{1 + j(\theta_x - \theta_s)\} \right] -$$

$$- I_{ss} \pi + \Delta I R + I_{ss}' j\omega M = 0$$

$$\Rightarrow I_{ss} \left[1 - \frac{R_s}{R_x} \{1 + j(\theta_x - \theta_s)\} \right] R + I_{ss}' (j\omega M - \pi) = 0$$

$$\Rightarrow I_{ss} \left[\pi - \frac{R_s R}{R_x} + j(\theta_x - \theta_s) \right] = I_{ss}' (\pi - j\omega M)$$

$$\Rightarrow \frac{R_s R}{R_m} \left[1 + j(\theta_m - \theta_s) \right] = \frac{R}{R_m} (\pi - j\omega M)$$

$$\Rightarrow R - \frac{R_s R}{R_m} = 0$$

$$\Rightarrow \left(1 - \frac{R_s}{R_m} \right) = \frac{R}{R_m}$$

$$\Rightarrow \frac{R_s}{R_m} = 1 - \frac{R}{R_m}$$

$$\Rightarrow R_m = \frac{R_s}{\left(1 - \frac{R}{R_m} \right)}$$

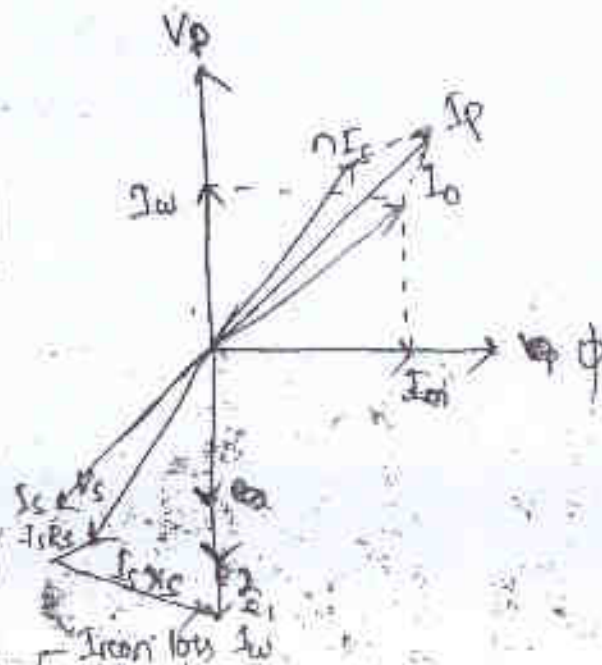
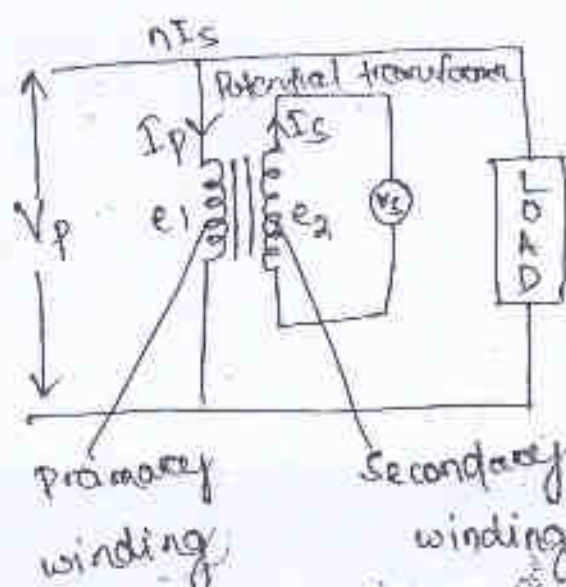
Now again $R - \frac{R_s R}{R_m} \left[1 + j(\theta_m - \theta_s) \right] = \frac{R}{R_m} (\pi - j\omega M)$ Imaginary

$$\Rightarrow \frac{R_s R}{R_m} (\theta_m - \theta_s) = \omega M$$

$$\theta_m - \theta_s = \frac{\omega M R_m}{R_s R}$$

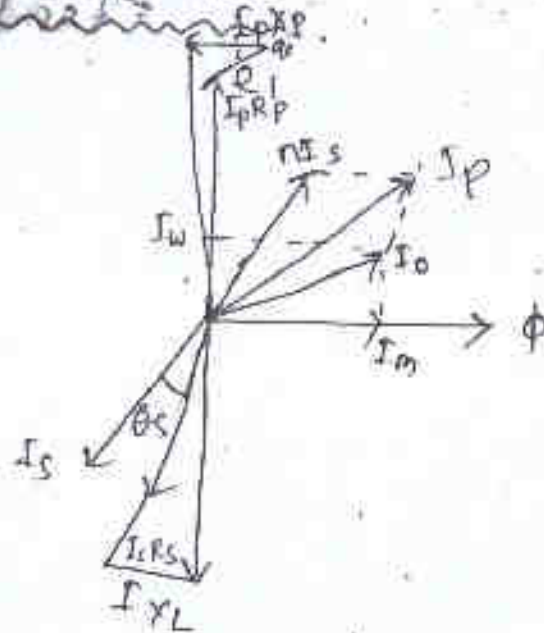
$$\Rightarrow \theta_s = \frac{\omega M R_m}{R_s R} + \theta_s$$

* Potential Transformer $n =$ transformation ratio



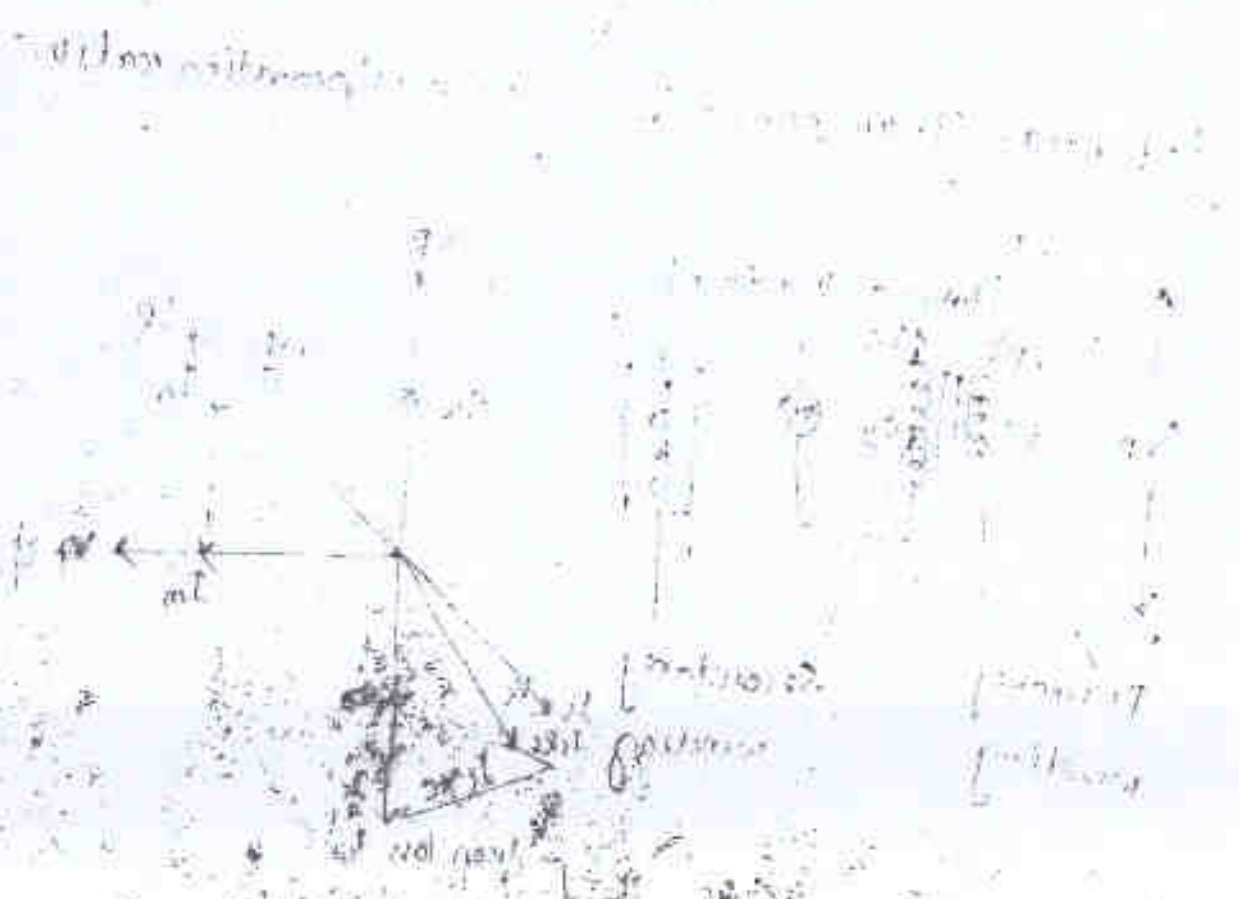
* Potential Transformer

DATE 13/02/15



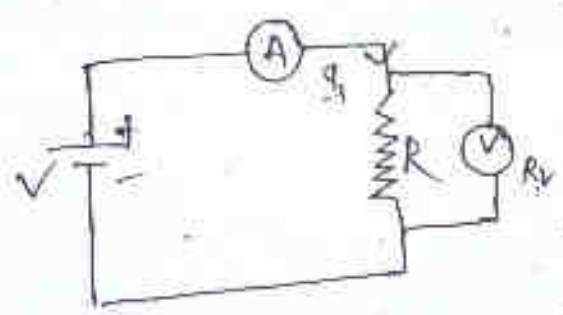
* Testing of potential Transformer:-

It is same as the silsbee's method



MEASUREMENT OF RESISTANCE

① Ammeter - Voltmeter method :-



$R = 1 \Omega$
 $R_V = 5 \Omega$
 $I = 100 \text{ mA}$
 $V = 100 \text{ mV}$

$I_1 = I \times \frac{S}{S+1}$

$= 100 \times 10^{-3} \times \frac{5}{6} = 0.0833 \text{ A}$

$V = 0.0833 \times 1 = 0.0833 \text{ V}$

$R_m = \frac{100 \text{ mV}}{100 \text{ mA}} = 1 \Omega$

$= \frac{0.0835}{0.1} = 0.833$

$\therefore \text{Error} = \frac{\text{Measured value} - \text{True value}}{\text{True value}}$

$= \frac{1 - 0.833}{1} = \pm 16.7\%$
 $\pm 20.04\%$

Date 18/02/15

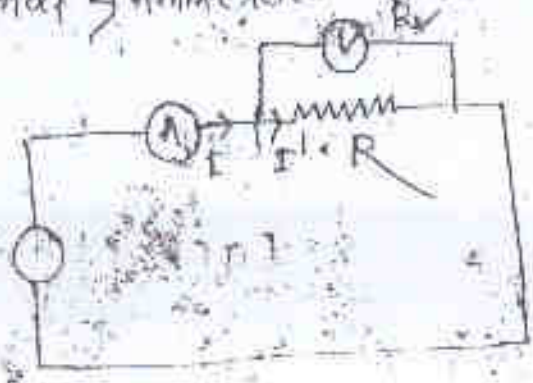
Low resistance $\rightarrow < 1 \Omega$

① Voltmeter Voltage drop method \rightarrow Ammeter - Voltmeter

$R_m = \frac{V}{I}$

$I' = I \frac{R_V}{R+R_V}$

$V = I' R = I \frac{R_V}{R+R_V} R$



$$R_m = \frac{R R_v}{R + R_v}$$

$$\Rightarrow R_m R + R_v R_m = R R_v$$

$$\Rightarrow R_v R_m = R (R_v - R_m)$$

$$\Rightarrow R = \frac{R_v R_m}{(R_v - R_m)}$$

Correction factor

$$K = \frac{R_v}{R_v - R_m}$$

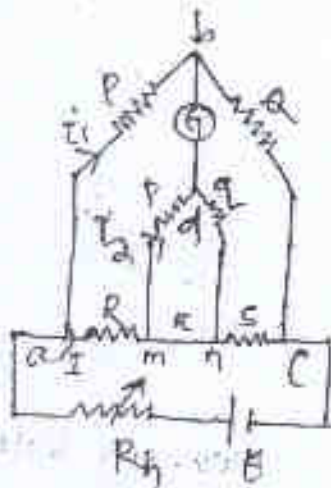
* Kelvin Double Bridge method :- X

Here,

R = Unknown Resistance

s = standard

P, Q, p, q = four non-inductive resistances



r = Resistance of the lead connecting R and s.

* when Galvanometer reading is $\frac{P}{Q} = \frac{p}{q}$ at that time the deflection shows zero.

* A sensitive 'D' Arsonval Galvanometer is connected across the dividing point of Pq and pq. Now calculate emf across a, b

$$E_{ab} = E_{ac} \frac{P}{P+Q}$$

$$E_{ac} = \left[R + \frac{(P+Q)r}{P+Q+r} + s \right] I$$



Calculate $E_{\text{amt}} = IR + E_{\text{mn}} \left(\frac{P}{P+Q} \right)$

Now, calculate E_{mn}

$$E_{\text{mn}} = IR + I \left(\frac{(P+Q)r}{P+Q+rc} \right) \frac{P}{P+Q}$$

$$= I \left[R + \frac{Pr}{P+Q+rc} \right]$$

Now, $E_{\text{ab}} = E_{\text{amt}}$

$$\Rightarrow E_{\text{ac}} \frac{P}{P+Q} = IR + E_{\text{mn}} \left(\frac{P}{P+Q} \right) \quad (5)$$

$$\Rightarrow \frac{P}{P+Q} \left[R + \frac{(P+Q)r}{P+Q+rc} + s \right] = I \left[R + \frac{Pr}{P+Q+rc} \right]$$

$$\Rightarrow \frac{P}{P+Q} \left[R + \frac{(P+Q)r}{P+Q+rc} + s \right] = \left[R + \frac{Pr}{P+Q+rc} \right]$$

$$\Rightarrow \frac{P}{P+Q} R - R = \frac{Pr}{P+Q+rc} - \left(\frac{P}{P+Q} \right) \left[\frac{(P+Q)r}{P+Q+rc} + s \right]$$

$$\Rightarrow \left(\frac{P-Q}{P+Q} \right) R = \frac{Pr}{P+Q+rc} - \frac{P}{P+Q} \left[\frac{(P+Q)r}{P+Q+rc} + s \right]$$

$$\Rightarrow \left(\frac{-Q}{P+Q} \right) R = \frac{Pr}{P+Q+rc} - \frac{P}{P+Q} \left[\frac{(P+Q)r}{P+Q+rc} + s \right]$$

$$\Rightarrow -R = \frac{P+Q}{Q} \left(\frac{Pr}{P+Q+rc} \right) - \frac{P}{Q} \frac{(P+Q)r}{P+Q+rc} - \frac{P}{Q} s$$

$$\Rightarrow -R = \left(\frac{P}{Q} + 1 \right) \left(\frac{Pr}{P+Q+rc} \right) - \frac{P}{Q} \frac{(P+Q)r}{P+Q+rc} - \frac{P}{Q} s$$

$$\Rightarrow -R = \frac{P}{Q} \frac{Pr}{P+Q+rc} + \frac{Pr}{P+Q+rc} - \frac{P}{Q} \left(\frac{Pr}{P+Q+rc} \right) - \frac{P}{Q} s$$

$$\frac{P}{Q} \frac{Qr}{P+Q+rc} = \frac{P}{Q} s$$

$$\Rightarrow R = \frac{P}{Q} s + \frac{Pr}{Q} \left(\frac{Qr}{P+Q+rc} \right) + \frac{Pr}{P+Q+rc}$$

$$\Rightarrow R = \frac{P}{Q} s + \frac{Pr}{Q} \left(\frac{Qr}{P+Q+rc} \right) + \frac{Pr}{P+Q+rc}$$

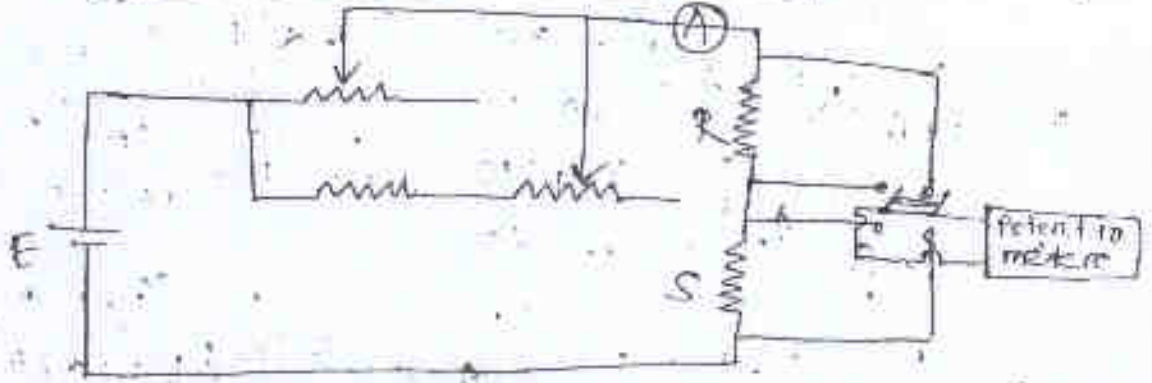
$$R = \frac{P}{Q} \times \frac{P+Q}{P} = \frac{P}{Q} \times \left(\frac{P}{Q} + \frac{P}{Q} \right) \rightarrow 0$$

taking, $\frac{P}{Q} = \frac{P}{Q}$ we will get

$$\Rightarrow R = \frac{P}{Q} S$$

② Potentiometer Method :-

→ Two pole Double Throw



In case 1 → potentiometer connect across S

$$V_S = I_S S \quad \text{--- (i)}$$

In 2nd case → potentiometer connect across R

$$V_R = I_R R \quad \text{--- (ii)}$$

Now, eqⁿ (i) ~~is~~ by dividing eqⁿ (ii)

$$\frac{V_S}{V_R} = \frac{I_S S}{I_R R}$$

$$\Rightarrow R = \frac{V_R}{V_S} S$$

low Resistance	→	0 - 1 Ω
Medium	→	1 - 10 kΩ
High	→	100 kΩ - above

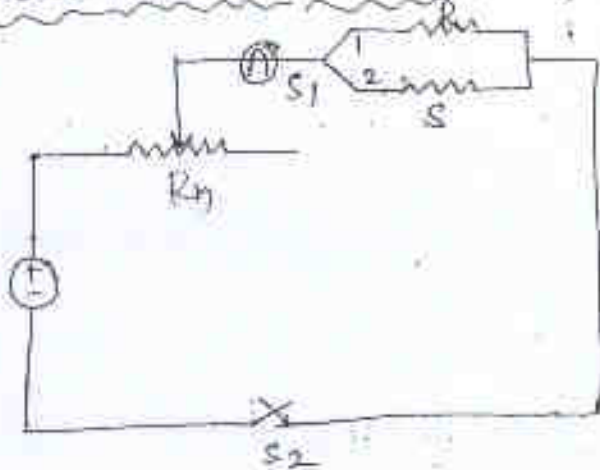
Medium Resistance ($1\Omega - 100k\Omega$)

Date 19/02/15

(i) Substitution method

(ii) Wheatstone bridge method

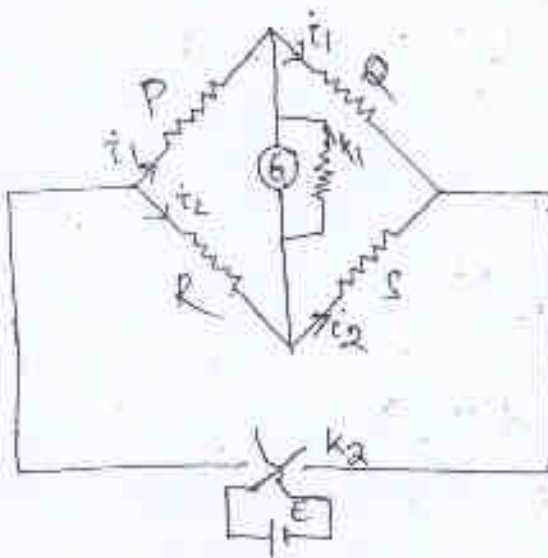
(i) Substitution method :-



$R = \text{Unknown Resistance}$

$S = \text{standard Resistance}$

(ii) Wheatstone bridge method



k_1 & k_2 are two switches

$$i_1 P = i_2 R \quad \text{--- (i)}$$

$$i_1 Q = i_2 S \quad \text{--- (ii)}$$

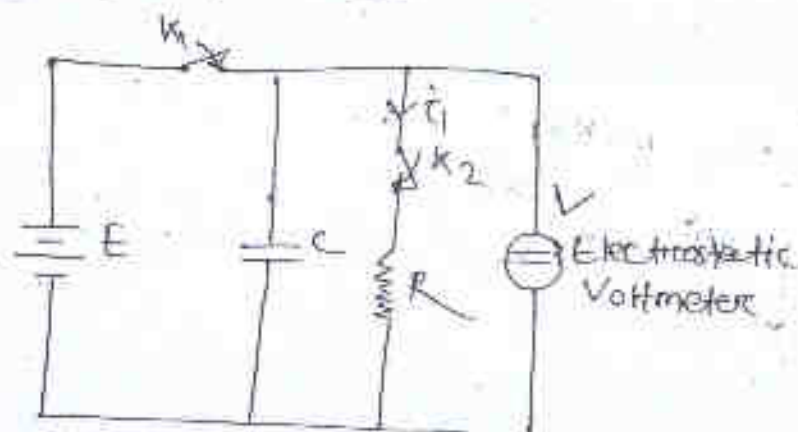
Now dividing eqⁿ (i) & (ii) we get

$$\Rightarrow \frac{i_1 P}{i_1 Q} = \frac{i_2 R}{i_2 S}$$

$$\Rightarrow \frac{P}{Q} = \frac{R}{S}$$

High Resistance (look at above)

Charge method :-



According to ohm's law $i_1 = \frac{V}{R}$

$$i_1 = -C \frac{dv}{dt} \quad (\text{at time } t)$$

For discharge capacitor

$$i_1 + C \frac{dv}{dt} = 0$$

$$\Rightarrow i_1 = -C \frac{dv}{dt} \quad (\text{at time } t)$$

$$\frac{V}{R} = -C \frac{dv}{dt}$$

$$\Rightarrow \int \frac{dv}{v} = \int -\frac{1}{RC} dt$$

$$\Rightarrow \log_e v = -\frac{t}{RC} + \log_e k$$

At $t=0$, $v=E$

$$\Rightarrow \log_e E = \log_e k$$

$$\Rightarrow \boxed{E = k}$$

$$\Rightarrow \log_e v = -\frac{t}{RC} + \log_e E \quad (\text{General Equation})$$

$$\Rightarrow \frac{t}{RC} = \log_e E - \log_e v$$

$$\Rightarrow \frac{t}{RC} = \log_e \left(\frac{E}{v} \right)$$

$$\Rightarrow \frac{t}{RC} = \frac{1}{\log_e \left(\frac{E}{v} \right)} \Rightarrow \boxed{R = \frac{t}{C \log_e \left(\frac{E}{v} \right)}}$$

Cable of length of 500 mtr is tested for insulation resistance by the loss of charge method. An electrostatic voltmeter of infinite resistance is connected between the cable conductor and the earth, forming a joint capacitance of 8×10^{-9} mf. It is observed that after charging the voltage falls from 350-150v in one minute. Calculate the insulation resistance of the cable of length 1000 mtr.

Soln:

Given data

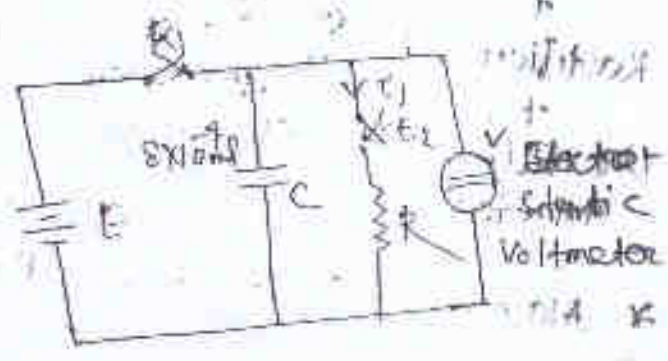
$L = 500 \text{ mtr}$

$C = 8 \times 10^{-9} \text{ mf}$

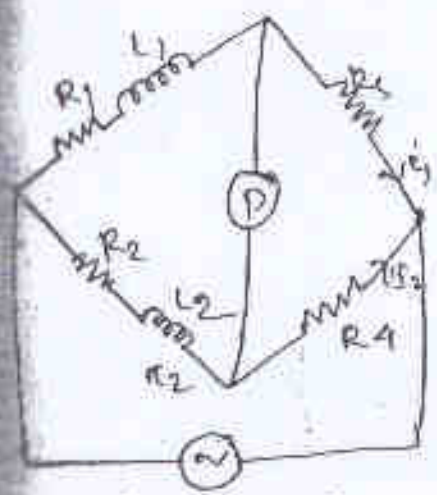
$t = 1 \text{ min} = 60 \text{ sec}$

$\Rightarrow R = \frac{t}{C \log_e \left(\frac{E}{V} \right)} \Rightarrow R = 8.851668759 \times 10^{10} \Omega$

$\Rightarrow R = 4.42 \times 10^{10} \Omega$



Measurement of inductance of Bridge:



Imaginary?

Real

$\Rightarrow Z_1 = Z_2 \times \frac{Z_3}{Z_4} \Rightarrow Z_1 = Z_2 \times \frac{R_3}{R_4}$

$\Rightarrow Z_1 = R_1 + j\omega L_1$

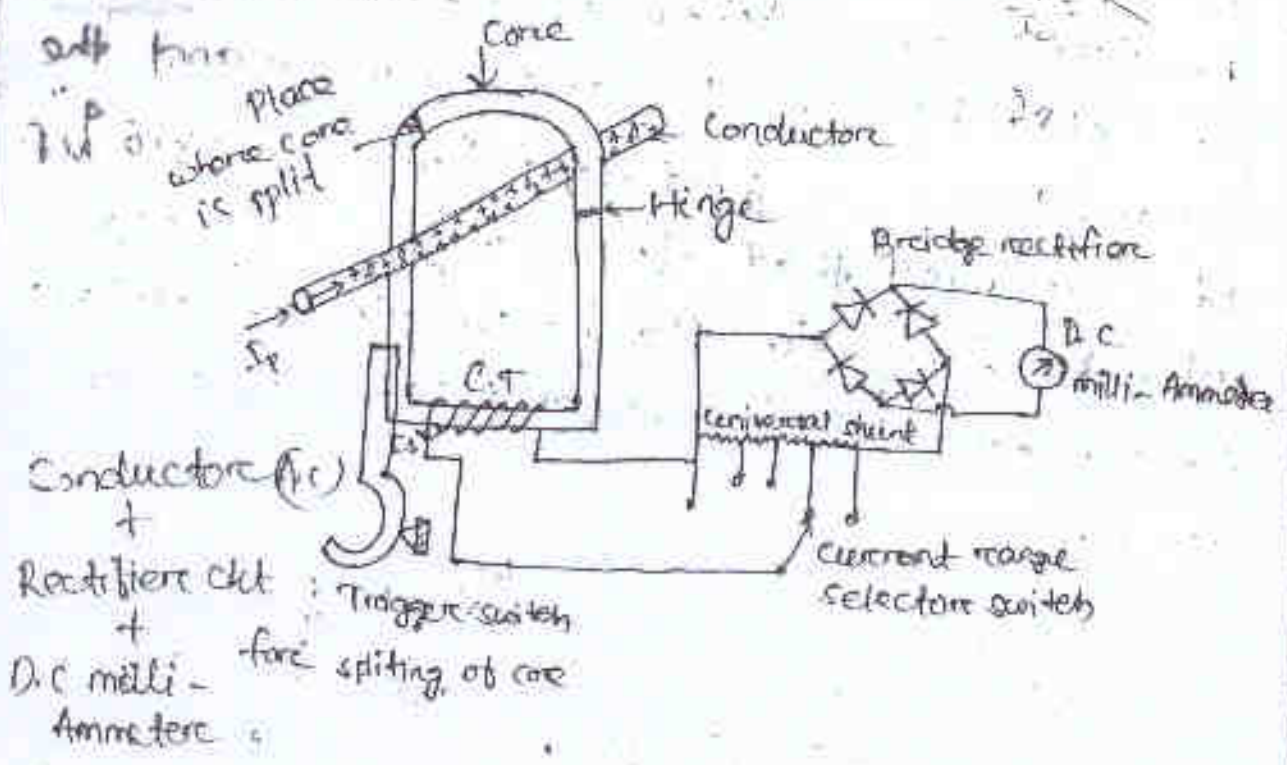
$Z_2 = (R_2 + j\omega L_2)$

$Z_3 = R_3$

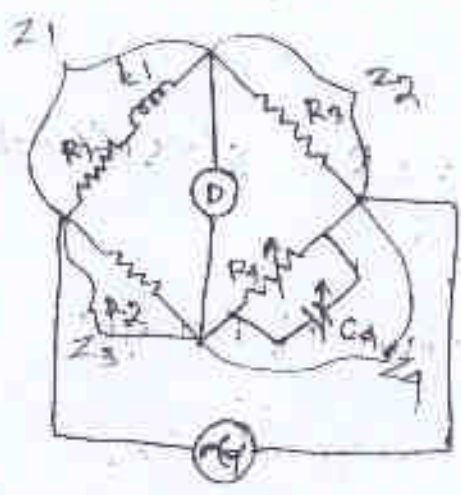
$\Rightarrow (R_1 + j\omega L_1) R_4 = (R_2 + j\omega L_2) R_3$
 $\Rightarrow R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega L_2 R_3$

$\Rightarrow R_1 R_4 = R_2 R_3 + R_3 \omega^2 L_2^2$
 $\Rightarrow R_1 R_4 = R_2 R_3 + R_3 \omega^2 L_2^2$

is construction



* Maxwell - Inductance - Capacitance bridge :-



$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

$$\Rightarrow Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L) \left[\frac{R_4 \times \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}} \right] = R_2 R_3$$

$$= (R_1 + j\omega L) \left[\frac{R_4}{j\omega C_4 R_4 + 1} \right] = R_2 R_3$$

$$R_1 R_4 + (j\omega L_1 R_4) = \frac{R_2 R_3}{R_4} + j\omega R_2 R_3 C_1 R_4 \quad \text{Imaginary}$$

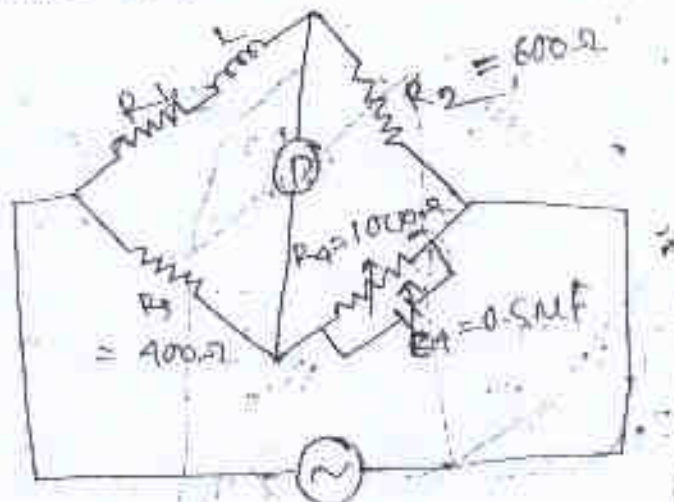
$$\Rightarrow R_1 R_4 = R_2 R_3$$

$$\Rightarrow R_1 = \frac{R_2 R_3}{R_4}$$

$$j\omega L_1 R_4 = j\omega R_2 R_3 C_1 R_4$$

$$\Rightarrow L_1 = R_2 R_3 C_1$$

Prob Determine the value of R and L of the inductor connected in the bridge circuit of balance has been obtained. Also determine the Q factor of the coil.



Solⁿ

Given data

$$R_1 = ? , L = ?$$

$$R_2 = 600 \Omega , R_3 = 400 \Omega , R_4 = 1000 \Omega$$

$$C_1 = 0.5 \mu F$$

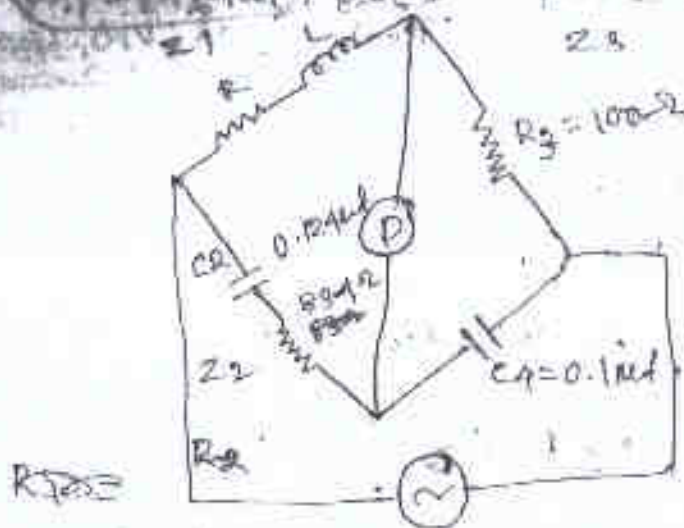
$$\therefore R_1 = \frac{R_2 R_3}{R_4} = \frac{600 \times 400}{1000} = 240 \Omega$$

$$L_1 = R_2 R_3 C_1 = 600 \times 400 \times 1000 = 0.12 H$$

$$\therefore \text{Quality factor} = \frac{\omega L_1}{R_1} = \frac{2\pi \times 7 \times 0.12}{240}$$

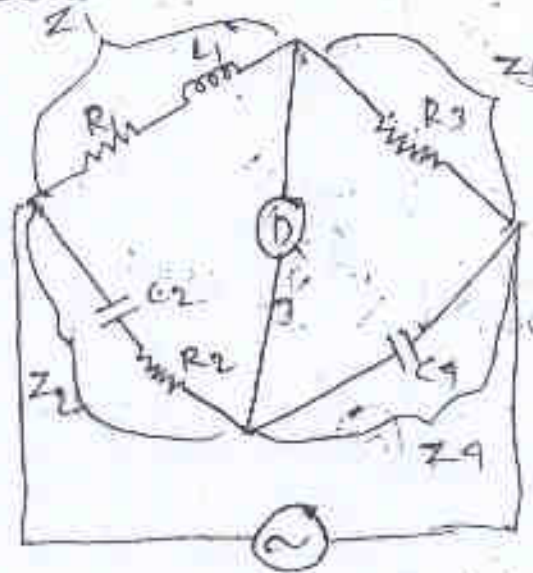
$$= \frac{2 \times 3.14 \times 7 \times 0.12}{240} = 5.14$$

original $Z_1 = R_1 + j\omega L_1$ $Z_3 = R_3$



$$R_1 = \frac{R_2 R_3}{R_4} = \frac{100 \times 80}{83}$$

* Owen's bridge:



$$Z_1 = R_1 + j\omega L_1$$

$$Z_2 = R_2 + \frac{1}{j\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = \frac{1}{j\omega C_4}$$

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

$$\Rightarrow Z_1 Z_4 = Z_2 Z_3$$

$$\Rightarrow (R_1 + j\omega L_1) \left(\frac{1}{j\omega C_4} \right) = \left(R_2 + \frac{1}{j\omega C_2} \right) R_3$$

$$\Rightarrow \frac{R_1}{j\omega C_4} + \frac{j\omega L_1}{j\omega C_4} = R_2 R_3 + \frac{R_3}{j\omega C_2}$$

$$\Rightarrow \frac{R_1}{j\omega C_4} + \frac{L_1}{C_4} = R_2 R_3 + \frac{R_3}{j\omega C_2}$$

Real part

$$\frac{L_1}{C_1} = R_2 R_3$$

$$\Rightarrow L_1 = R_2 R_3 C_1$$

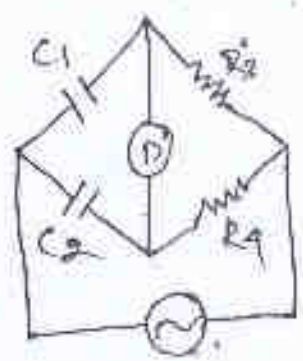
Imaginary part

$$\frac{R_1}{j\omega C_1} = \frac{R_3}{j\omega C_2}$$

$$\Rightarrow R_1 = \frac{R_3 C_1}{C_2}$$

* Measurement of Capacitance

① De Sauty's Bridge Method

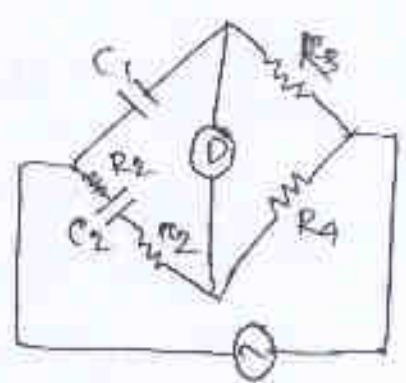


C_1 = Unknown Capacitance
 C_2 = standard Capacitance

$$\frac{1}{j\omega C_1} R_4 = \frac{1}{j\omega C_2} R_3$$

$$\Rightarrow C_1 = \frac{R_4}{R_3} C_2$$

* Modify of the De Sauty's Bridge method



C_1 = Unknown Capacitance
 C_2 = Standard Capacitance

$$\frac{1}{j\omega C_1} R_4 = \left(\frac{1}{j\omega C_2} + R_2 + R_3 \right) R_3$$

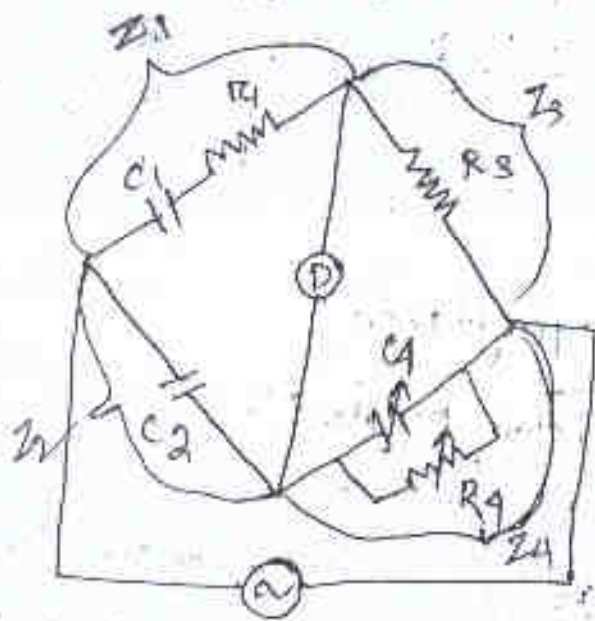
$$\frac{R_4}{j\omega C_1} = \frac{R_3}{j\omega C_2} + R_2 R_3 + R_3^2$$

$$\Rightarrow \frac{R_4}{j\omega C_1} = \frac{R_3 + j\omega C_2 R_2 R_3 + j\omega C_2 R_3^2}{j\omega C_2}$$

$$\Rightarrow \frac{R_4 C_2}{R_3 + j\omega C_2 (R_2 R_3 + R_3^2)} = C_1$$

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Date 2/02/2015



$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

$$\Rightarrow Z_1 Z_4 = Z_2 Z_3$$

$$\Rightarrow R_1 + \frac{1}{j\omega C_1} \left(\frac{R_4 + \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}} \right) = \frac{R_3}{j\omega C_2}$$

$$\Rightarrow \left(R_1 + \frac{1}{j\omega C_1} \right) = \frac{\frac{R_4 \times 1}{j\omega C_4}}{\frac{R_4 + \frac{1}{j\omega C_4}}{j\omega C_4}}$$

$$\Rightarrow R_1 R_4 + \frac{R_4}{j\omega C_1} = \frac{R_3}{j\omega C_2} + \frac{R_3 C_4 R_4}{C_2}$$

$$\Rightarrow R_1 R_4 = \frac{R_3 R_4 C_1}{C_2}$$

$$\Rightarrow R_1 = R_3 \left(\frac{C_1}{C_2} \right)$$

$$\Rightarrow \frac{R_4}{C_1} = \frac{R_3}{C_2}$$

$$\Rightarrow C_1 = \frac{R_4 C_2}{R_3}$$

⑨ DIGITAL MULTIMETER

Main Function

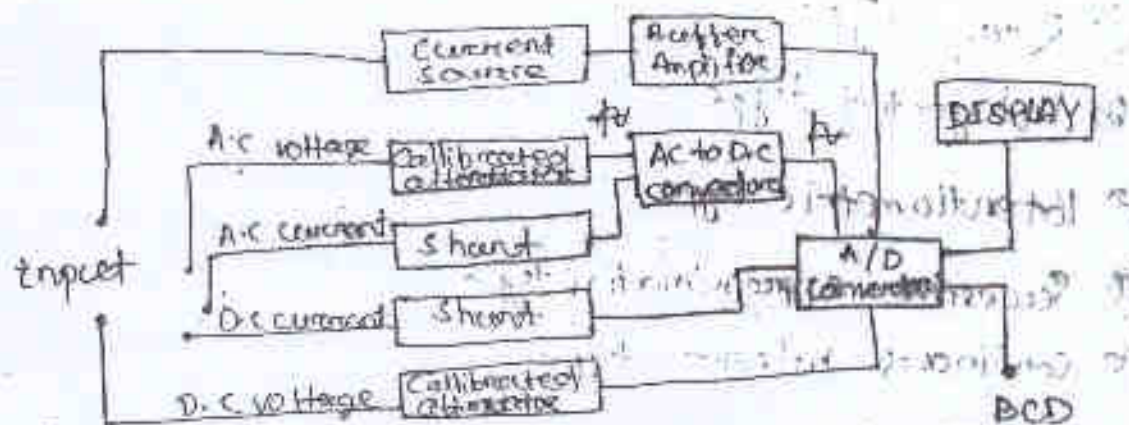
- A.C voltage
- D.C voltage
- A.C current
- D.C current
- Resistance

Additional functions

- Capacitance
- Temperature
- Forward biased diode drop
- Continuity
- Frequency

* Digital multimeter

Ripple reduction filter



Opposite of amplifier is calibrated attenuator

$$\text{ripple factor} = \frac{\text{RMS value of A.C}}{\text{Absolute of D.C O/P}}$$

BCD \rightarrow Binary coded Display Decimal

VSWR \rightarrow Voltage standing wave Ratio

Advantages Over a Analog instrument:

- ① Reduce observational errors, parallax, approximation
- ② Speed of experiment / operation.
- ③ Data can be send to other memory & storage devices.

* Digital Voltmeter

mainly the digital voltmeter is divided into

5 types - i.e. -

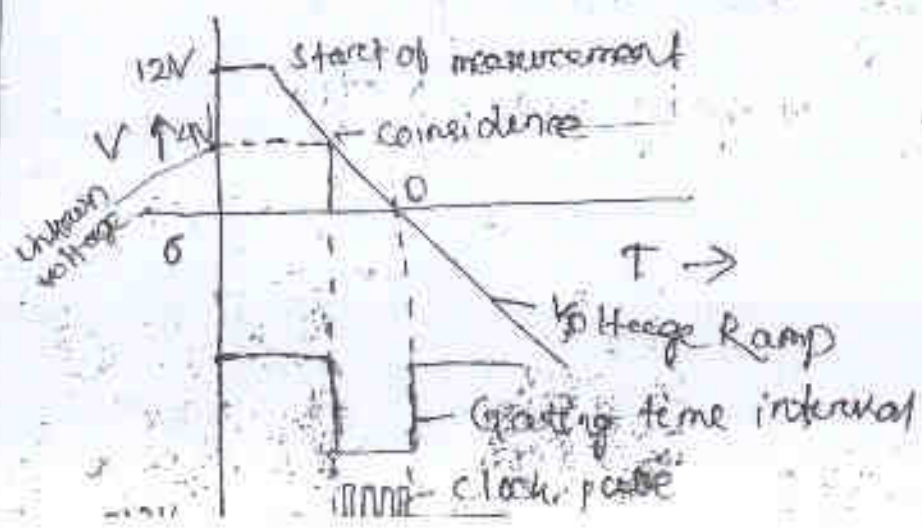
- ① Ramp type
- ② Integrated type
- ③ Potentiometric type
- ④ Successive approximation type
- ⑤ Continuous balance type

① Ramp type

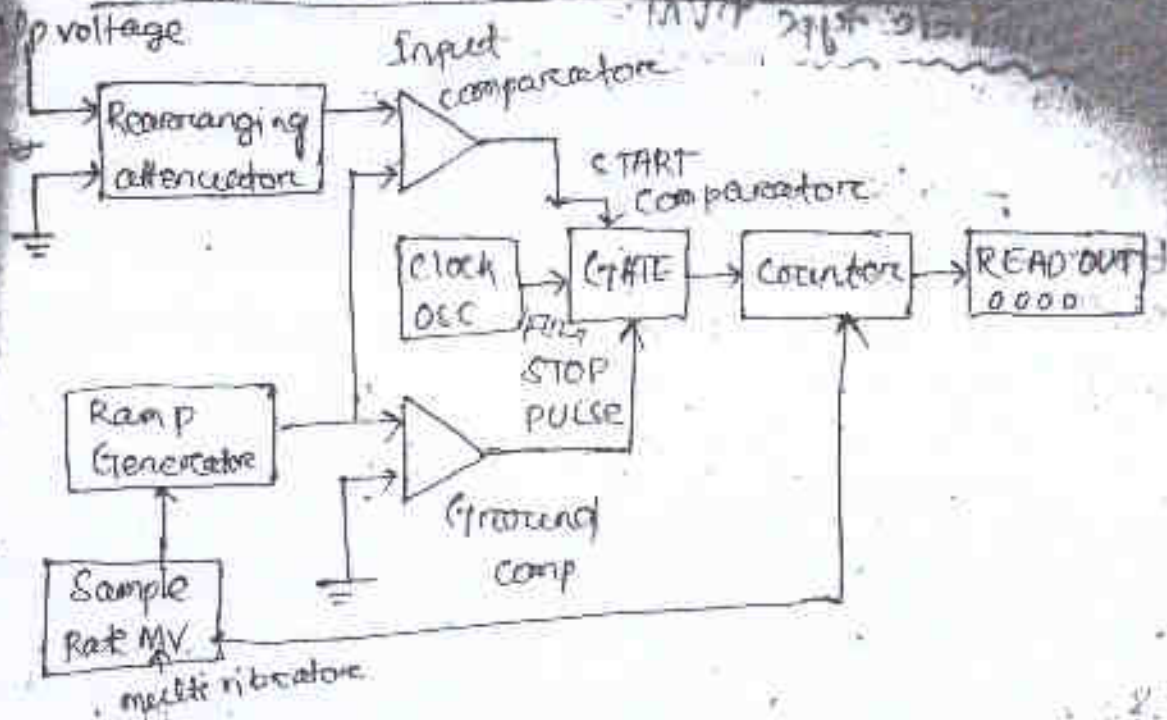
(counter → time → pulse → count)

Voltage measure in terms of time.

i/p voltage $\xrightarrow{\text{time}}$ o/voltage



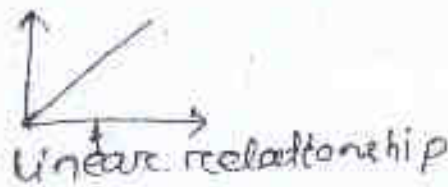
BLOCK DIAGRAM OF RAMP TYPE DVM



Ramp-function

$$t = 0 \quad t < 0$$

$$= kt \quad t > 0$$

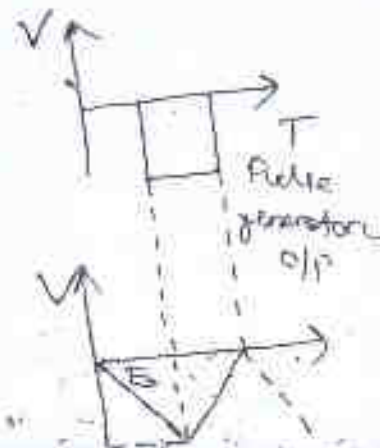
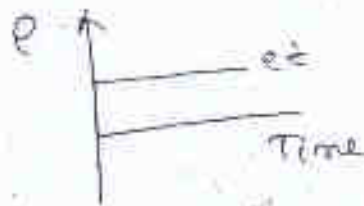
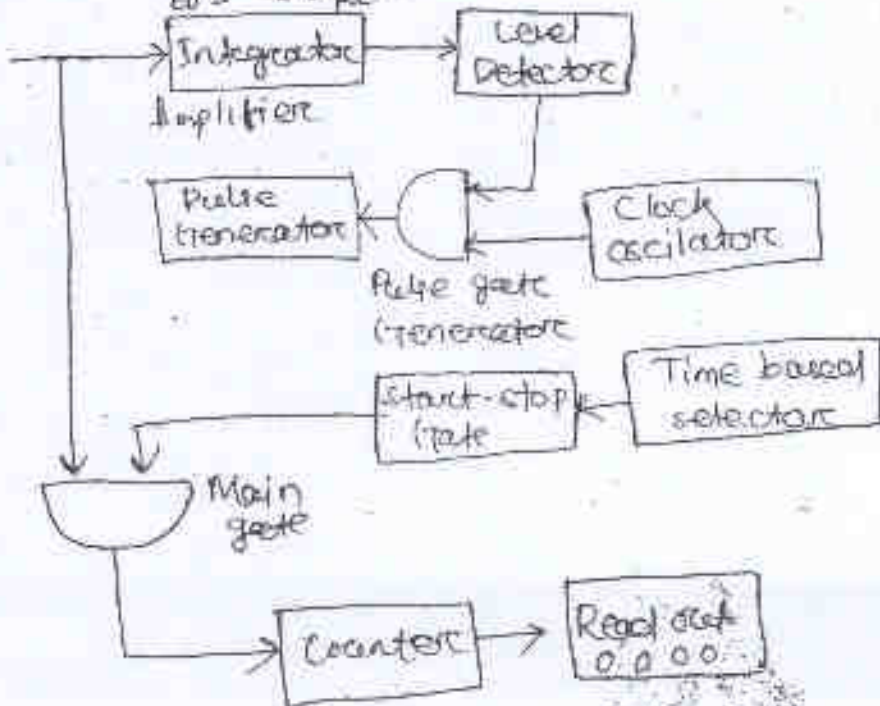


Date 26/02/15

② Integrating type DVM:-

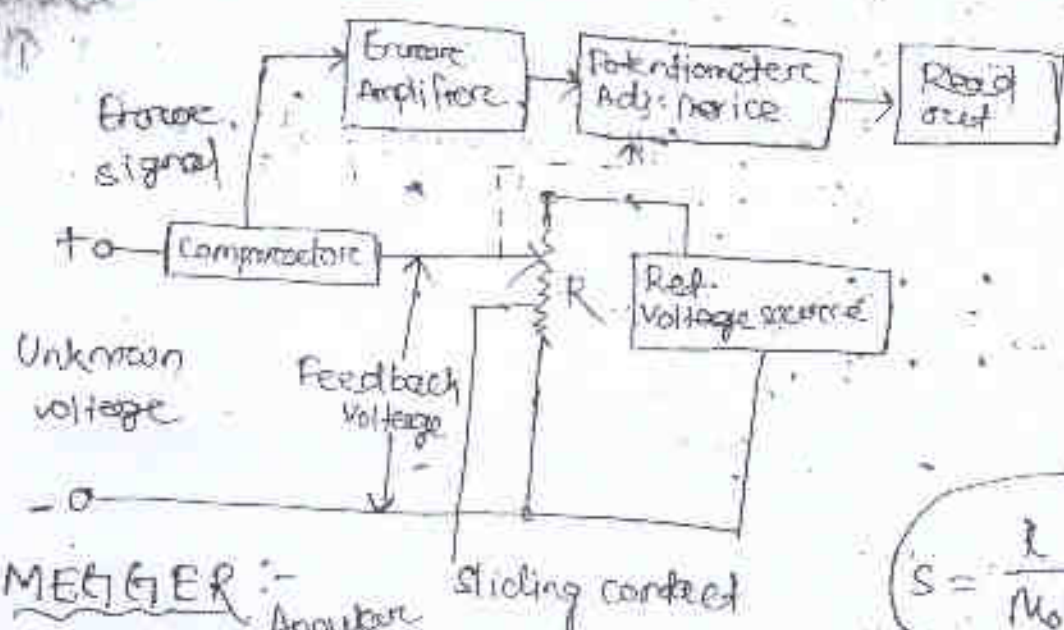
Voltage - frequency (V/F)

$$E_0 = -E_i \frac{t}{RC}$$

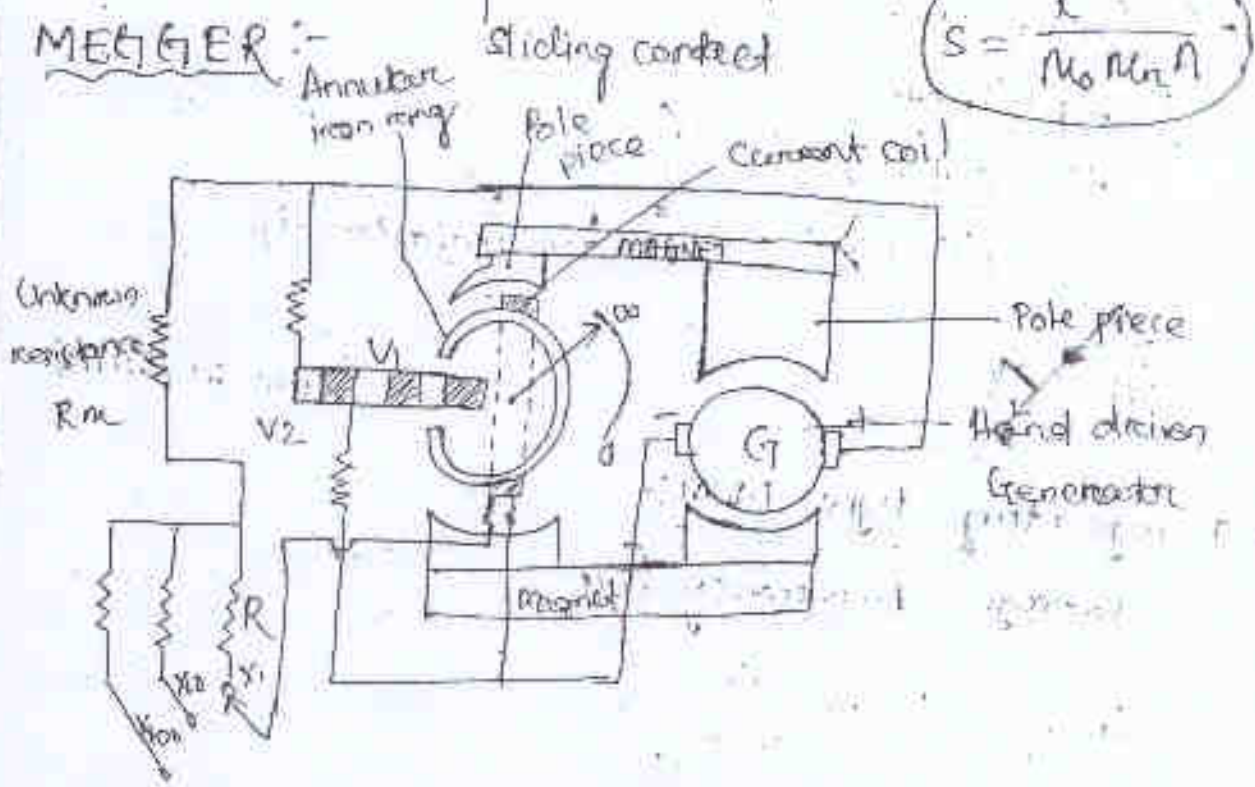


level at which detector o/p is produced

Operational amplifier type DVM:-



METGER



$$S = \frac{I}{M_0 M_1 A}$$

⑦ Measurement of Resistance

① 1000

* Classification of resistance: (2 marks)

① Low resistance - This include all resistance of the order of 1Ω and under.

→ Such resistance may be met within the armature and series winding, in ammeters shunt, cable length, contacts etc.

② Medium resistance - This class include resistance from about 1Ω upward to about $100 k\Omega$.

→ Majority of electrical apparatus used in these range.

③ High resistance - Resistance of order of $100 k\Omega$ and upwards.

→ Insulation resistance of components.

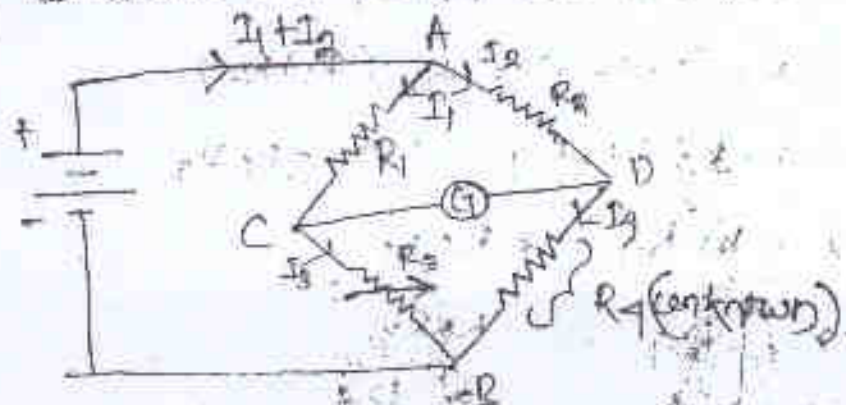
→ Resistance of high resistance.

→ Volume resistivity and surface resistivity.

④ Wheatstone bridge method (Long Question)

→ It is the simplest form of bridge, for the purpose of the measurement of resistance.

→ It is the most accurate method available for measuring resistance and is popular for laboratory use.



Basic Operation of Wheatstone Bridge

→ The bridge has four resistive arms, together with a source of EMF and null detector (galvanometer).

→ The current through the galvanometer depends on the potential difference between the point C and D.

→ The bridge is said to be balanced when the potential difference across the galvanometer is '0' volt. So, that there is no current through the galvanometer. Hence the bridge is balanced when potential difference 'c' and 'd' is equal.

$$I_1 R_1 = I_2 R_2 \quad \text{--- (i)}$$

When the current through the galvanometer is zero. The following condition should be satisfied.

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \quad \text{--- (ii)}$$

$$I_2 = I_4 = \frac{E}{R_2 + R_4} \quad \text{--- (iii)}$$

Substituting the value of I_1 and I_2 in eqⁿ (i)

$$\frac{E}{R_1 + R_3} R_1 = \frac{E}{R_2 + R_4} R_2$$

$$\Rightarrow \frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$

$$\Rightarrow R_1 (R_2 + R_4) = R_2 (R_1 + R_3)$$

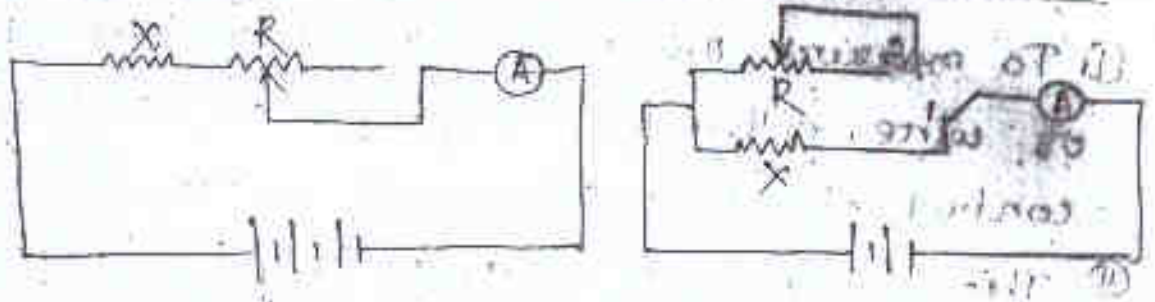
$$\Rightarrow R_1 R_2 + R_1 R_4 = R_2 R_1 + R_2 R_3$$

$$\Rightarrow R_1 R_4 = R_2 R_3$$

$$\Rightarrow R_4 = \frac{R_2 R_3}{R_1}$$

Measurement of Medium Resistance \rightarrow (3)

(1) Substitution method :- Substitution of resistances



Let R be a variable resistance which can be changed in small steps say of 0.1Ω .

- \rightarrow In first figure first resistance X is put in the circuit and the value of the current is noted.
- \rightarrow Then resistance X is removed and it is substituted by a known, variable resistance R which is varied so that the value is equal to the unknown resistance.

If $R =$ a fixed value, then the ammeter reading will be for (i) resistance X and R in series, $I_1 = \frac{V}{R+X}$

(ii) When resistance X is removed $I_2 = \frac{V}{R}$

$$\therefore \frac{I_2}{I_1} = \frac{R+X}{R} = 1 + \frac{X}{R}$$

$$\Rightarrow \frac{X}{R} = 1 - \frac{I_2}{I_1}$$

$$\Rightarrow X = R \left(\frac{I_2 - I_1}{I_1} \right)$$

\rightarrow In second figure 'two-way' switch first make contact with 1 and then 2 and let these readings be I_1 and I_2 .

$$I_1 = \frac{V}{X}$$

$$I_2 = \frac{V}{R}$$

$$\Rightarrow \frac{I_2}{I_1} = \frac{X}{R}$$

$$\Rightarrow X = \frac{I_2}{I_1} R$$

R_1, R_2, R_3 Ratio arm; R_x = standard arm
Application of Wheatstone bridge

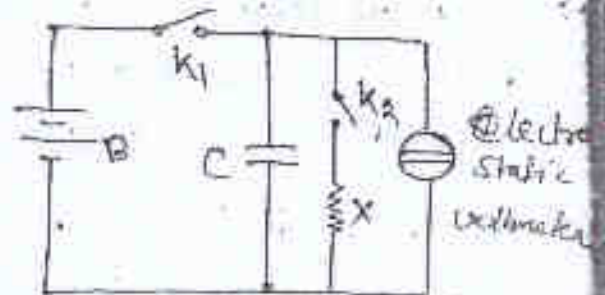
- ① To measure D.C. resistance of ^{variable type} wire, either for the purpose of quality control or to calibrate one of some other.
- ② This bridge is used extensively by telephone companies and others to locate cable fault.

* Measurement of High Resistance.

* Loss of charge method - [Long Question]

→ This method is used for

the measurement of very high insulation resistance.



→ With key K_1 closed and

K_2 open capacitor is charged up to a suitable voltage. Then the capacitor is allowed to discharge through the unknown resistance X by opening K_1 & closing K_2 . The terminal voltage is being observed for a long time.

V = terminal voltage, Q = charge in coulomb

then current through the resistance X

$$i = \frac{dQ}{dt} = -C \frac{dV}{dt}$$

$$\text{But } i = \frac{V}{X}$$

$$\therefore \frac{V}{X} = -C \frac{dV}{dt}$$

$$\Rightarrow C \frac{dV}{dt} + \frac{V}{X} = 0$$

$$\frac{dV}{V} = -\frac{1}{X} dt$$

exam (A)

Integrating both side we get

$$\log_e v = -\frac{t}{cx} + \log_e k$$

Let, $v = E$ when $t = 0$

then $k = k \cdot \log_e v = \log_e E - \frac{t}{cx}$

or $\frac{t}{cx} = \log_e \frac{E}{v}$

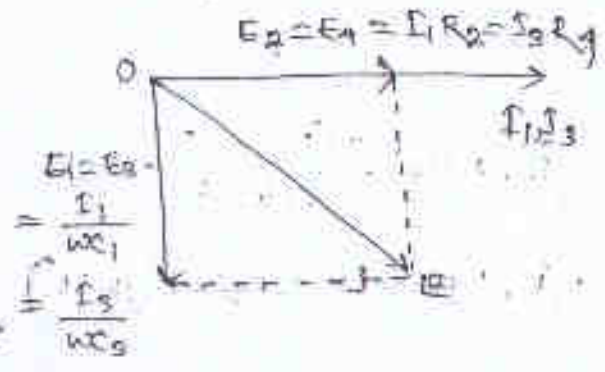
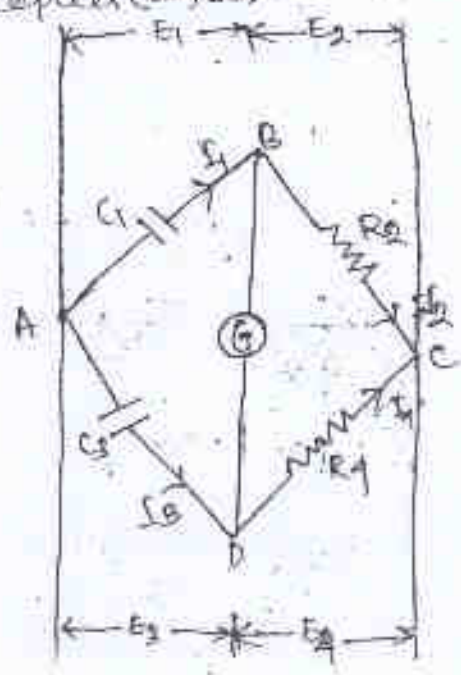
or $x = \frac{t}{c \log_e \frac{E}{v}} = \frac{0.4242t}{c \log_{10} \frac{E}{v}}$ ohm.

types
by
phone

Measurement of Capacitance

De Sauty Bridge method

The bridge is the simplest method of comparing two capacitances.



Let C_1 = Capacitor whose capacitance is to be measured.
 C_3 = Standard capacitor.

R_2, R_4 = Non inductive resistance

At balance $I_1 = I_2, I_3 = I_4$

and $\left(\frac{1}{j\omega C_1}\right) R_4 = \left(\frac{1}{j\omega C_3}\right) R_2$

Electro
Static
Voltmeter

able
is charge
of k_1 &
ranged

comb

Advantages :- The bridge is quite simple and provides easy calculation. (6)

Disadvantages :- The bridge gives inaccurate results for imperfect capacitors (the capacitors which are not free from dielectric loss). Hence we can use this bridge only for comparing perfect capacitors.

* Measurement of Inductance :-

Maxwell bridge Method :-

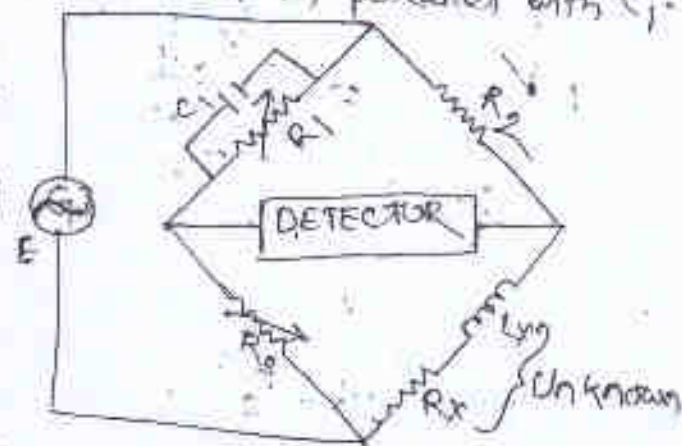
Maxwell bridge measures an unknown inductance in terms of a known capacitor.

- The use of standard arm offers the advantages of compactness and easy shielding.
- The capacitor is almost a loss-less component.
- One arm has a resistance R_1 in parallel with C_1 .

$$\therefore (R_x + j\omega L_x) \left(\frac{R_1}{1 + j\omega C_1 R_1} \right) = R_2 R_3$$

$$\Rightarrow R_1 R_x + j\omega R_1 L_x = R_2 R_3 (1 + j\omega C_1 R_1)$$

$$\Rightarrow R_1 R_x + j\omega R_1 L_x = R_2 R_3 + j\omega C_1 R_1 R_2 R_3$$



Equating imaginary and real quantity $R_1 R_x = R_2 R_3 \Rightarrow$

$$R_x = \frac{R_2 R_3}{R_1}$$

$$\therefore j\omega R_1 L_x = j\omega C_1 R_1 R_2 R_3$$

$$\Rightarrow L_x = C_1 R_2 R_3$$

Quality factor $Q = \frac{\omega L_x}{R_x} = \frac{\omega C_1 R_2 R_3}{(R_2 R_3 / R_1)} = \frac{\omega C_1 R_2 R_3}{R_2 R_3} \times R_1$

$$Q = \omega C_1 R_1$$

Advantages →

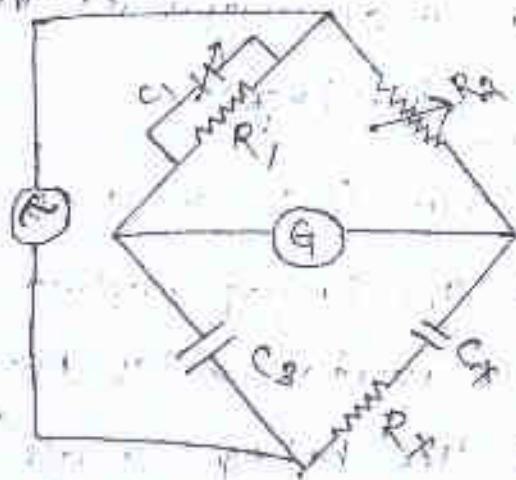
- ① The balance eqⁿ is independent of losses.
- ② The measurement is independent of excitation frequency.
- ③ The scale of resistance can be calibrated to read inductance directly.
- ④ A wide range of inductance at power and audio frequency can be measured.

Schering Bridge:

It is one of the most important A.C bridge, widely used for the measurement of unknown capacitors, dielectric loss and power factor.

- The capacitor C_3 is a high quality mica capacitor (less loss) or air capacitor (having very stable value).

The general eqⁿ for balance is



$$Z_1 Z_x = Z_2 Z_3$$

$$\left(\frac{R_1 \times \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} \right) \times \left(R_x + \frac{1}{j\omega C_x} \right) = R_2 \times \frac{1}{j\omega C_3}$$

$$\Rightarrow \frac{R_1}{j\omega C_1 R_1 + 1} \times \frac{R_x j\omega C_x + 1}{j\omega C_x} = \frac{R_2}{j\omega C_3}$$

$$\Rightarrow \frac{R_1}{j\omega C_x} (R_x j\omega C_x + 1) = \frac{R_2}{j\omega C_3} (1 + j\omega C_1 R_1)$$

$$\Rightarrow \frac{R_1 R_x j\omega C_x}{j\omega C_x} + \frac{R_1}{j\omega C_x} = \frac{R_2}{j\omega C_3} + \frac{j\omega C_1 R_1 R_2}{j\omega C_3}$$

$$\Rightarrow R_1 R_x + \frac{R_1}{j\omega C_x} = \frac{R_2}{j\omega C_3} + \frac{j\omega C_1 R_1 R_2}{C_3} \quad (2)$$

Comparing imaginary and real quantity of both side

$$R_1 R_x = \frac{C_1 R_1 R_2}{C_3}$$

$$\Rightarrow R_x = \frac{C_1 R_2}{C_3}$$

and $\frac{R_1}{j\omega C_x} = \frac{R_2}{j\omega C_3}$

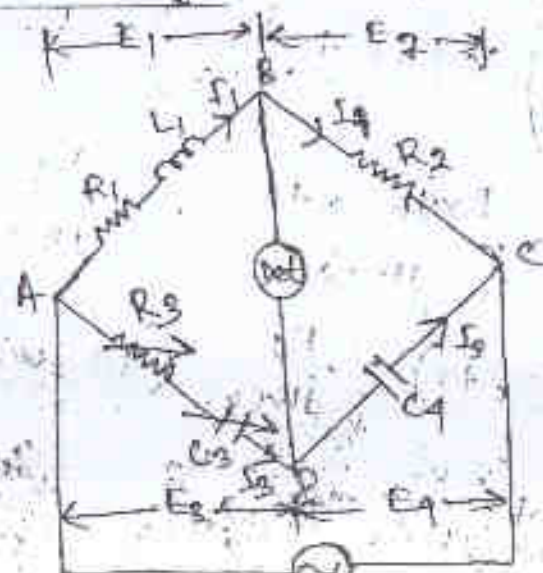
$$\Rightarrow C_x = \frac{R_1 C_3}{R_2}$$

→ The Schering bridge is widely used for testing very small capacitors at low voltages with high precision.

Disadvantages

- ① High Q value capacitor can't be measured.
- ② The bridge requires a variable standard capacitor which may be very expensive.
- ③ The bridge is also not suitable for the measurement of inductance of low Q value.

Owen bridge:-



9) The bridge may be used for measurement of an inductance in terms of capacitance.

L_1 = Unknown self-inductance of resistance R_1

R_2 = Fixed non-inductive resistance

R_3 = Variable non-inductive resistance

C_3 = Variable standard capacitor

C_4 = Fixed standard capacitor

$$(R_1 + j\omega L_1) \times \frac{1}{j\omega C_4} = R_2 \times \left(R_3 + \frac{1}{j\omega C_3} \right)$$

$$\Rightarrow \frac{R_1}{j\omega C_4} + \frac{L_1}{C_4} = R_2 R_3 + \frac{R_2}{j\omega C_3}$$

Equating real and imaginary quantity

$$\frac{R_1}{j\omega C_4} = \frac{R_2}{j\omega C_3} \Rightarrow R_1 = \frac{R_2 C_4}{C_3}$$

$$\text{and } \frac{L_1}{C_4} = R_2 R_3 \Rightarrow L_1 = R_2 R_3 C_4$$

Advantages →

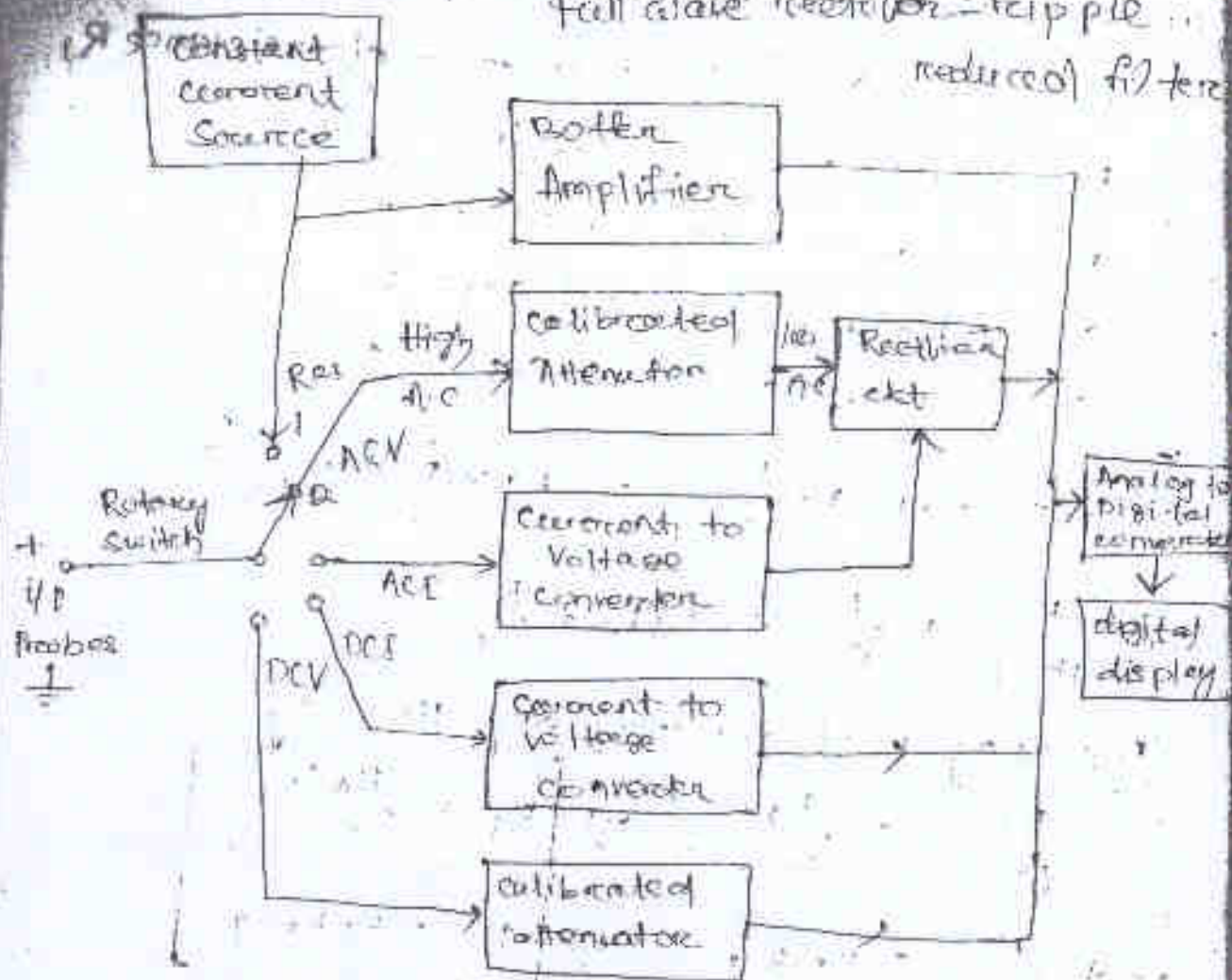
- 1) The balance eqⁿ are quite simple and don't contain any frequency component.
- 2) Since R_3 and C_3 both variable elements are in same arm. Convergence to balance condition is much more easy.
- 3) The bridge can be used over a wide range of measurement of inductance.

Disadvantages →

- 1) While measuring high Q value, the value of C_3 tends to be very large.
- 2) The variable capacitor is quite expensive.

Digital Multimeter (10)

Full wave rectifier principle
reduced filter



Multimeter is an instrument through which we can measure

- (i) Resistance
- (ii) Voltage current (D.C)
- (iii) Voltage current (A.C)
- (iv) Continuity etc.

→ In digital multimeter the measurement shows in digits (seven segment) whereas in analog by pointer on a calibrated scale.

→ In digital multimeter the reading is illuminate fig. which one can easily take the reading by digital multimeter.

→ The display may be LCD or LED so we can take the reading at distance also.

It has two probes. We cannot that. power (11)
whose measurement is to be read.

Error in energy metering and compensation

Phase error:- Normally the flux due to shunt magnet does not lag behind the supply voltage by exactly 90° due to the fact that the coil has some resistance. So, torque is not zero at zero power factor. This is called phase error. This error is compensated by means of adjustable copper bands placed over the central limb of the shunt magnet.

② Speed error:- An error in the speed of the motor when tested on the non-inductive load can be eliminated by correctly adjusting the position of the brake magnet.

③ Frictional Error:- Frictional forces at the rotor bearings and in the register mechanism give rise to an unwanted braking torque on the disc rotor. This can be reduced by making the ratio of shunt magnet blank and series magnet blank large with the help of two shading bands.

④ Creeping:- The slow but continuous rotation of motor when only the voltage i.e. the pressure coil is excited but no current flowing in the ckt is called creeping. To overcome the creeping on no load two holes are drilled in the disc, on the

(5) Temperature Error: The error due to temp⁽¹²⁾ variations of the various instruments are usually small because the various effects produced tend to neutralise one another.

(6) Overload Compensation: Under load conditions, the disc revolves continuously in the field of the series magnet and therefore there is a dynamically induced emf in the disc because of this rotation. This emf causes eddy currents which interact with the field of the series magnets to produce a braking torque. Thus at high values of load current the registration tends to be lower than the actual. To minimise the self-braking action, the full load speed of the disc is kept as low as possible.

(7) Voltage Compensation: Voltage variation may introduce errors due to non-linear magnetic characteristics of the shunt magnet core and also due to self-braking torque which is approximately proportional to the square of the supply voltage. Compensation for variations in supply voltage is provided by making use of a saturable magnetic shunt which diverts a greater proportion of the flux into the active path when the voltage rises.

Advantages of M.I Instruments!

- (i) They are used both for A.C. & D.C. measurement.
- (ii) Constructionally they are simple & robust.
- (iii) They have high operating torque.
- (iv) They have less frictional error.
- (v) Accuracy & precision are good.
- (vi) Overload capacity is also better.

Disadvantages of M.I instruments:

- (i) The scale is not uniform i.e., non-linear.
- (ii) Power consumption is high for low voltage range.
- (iii) As they are operated with A.C., there will be serious errors in measurement due to frequency.
- (iv) Also hysteresis errors and stray magnetic fields are prominent.

* Why an ammeter should be of very low resistance?

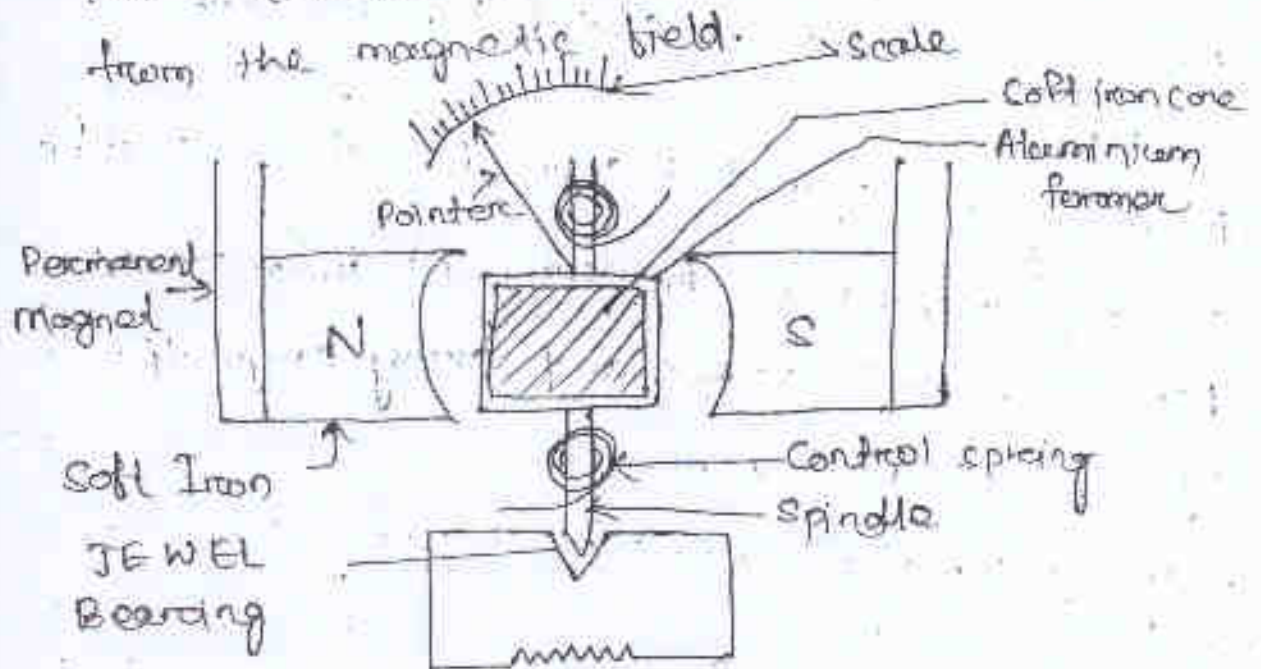
Ans:- When it is connected in series with any circuit, it does not change the current. Consequently, there is a small voltage drop and small power is absorbed. That's why an ammeter should be of very low resistance.

$$\text{Correction factor} = \frac{K_{\text{actual}}}{K_{\text{nominal}}}$$

Moving coil instrument :- There are two types of moving coil instrument :- (i) Permanent magnet type (ii) Dynamometer type

① Permanent Magnet Moving Coil (PMMC) →

Working principle :- The operation of a permanent magnet moving coil instrument is based upon a principle that when a current carrying conductor is placed in the magnetic field, a force acts on the conductor which tends to push it away from the magnetic field.



Construction

- (i) The fixed system of the instrument consists of a permanent magnet and a pair of sector shape soft iron pole piece drilled to the poles of the permanent magnet.
- (ii) The utility of the soft iron pole piece is to focus the magnetic field of permanent magnet between the air space and the coil.

The moving body system of the instrument (15)

consists of a spindle with a control spring, a soft iron core, aluminium former are mounted.

(i) The function of soft iron core is to -

- (a) Make the field radial and uniform
- (b) To decrease reluctance.

(ii) A copper wire that carries a current to be measured is mounted on the aluminium former.

(iii) The control spring on a PMMC instruments have dual utility -

- (a) They produce controlling torque
- (b) They lead the current to be measured

(iv) The aluminium pointer is attached to the rotating coil and the pointer moves around the calibrated scale indicates the deflection of the coil.

(v) The coil set cap is supported on jewel bearing in order to achieve free movement.

Deflecting torque \rightarrow

When the current passes through the coil, forces acts upon both it's sides and produces a deflecting torque:

Let, B = flux density in wb/m^2

l = length of the coil in m.

b = breadth of the coil in m.

N = No. of turns in the coil

I = current passing through the coil.

Magnitudes of force expressed by each side
The equation is given as $(F \propto BIL)$

For N turns, $\therefore T_d = NBIL \times d$
(for) \times (perpendicular distance)

$$T_d = NBIL (L \times b) \\ = NBA$$

As N, B, A are constant

$$T_d \propto I \text{ or } T_d = KI$$

* Advantages of Rectifier instruments \rightarrow

- (i) The frequency range extend from 20 Hz to high radio frequency.
- (ii) They have a practically uniform scale for most ranges.
- (iii) Low operating value of current.
- (iv) Accuracy is more.
- (v) Cost is low as compared to electro-dynamometer ^{type}.
- (vi) Less delicate as compared to thermocouple instrument.

Disadvantages \rightarrow

- (i) Loading effect is more, ~~seen~~
- (ii) Sensitivity is less.

Errors :-

initially assume resistors (10)

- (i) Effect of input wave form: Rectifier type instrument is calibrated in terms of RMS value of voltage & current. The problem is that the input wave form may or may not have same form factor on which the scale of these meters is calibrated.
- (ii) There may be some error due to the rectifier ckt as we not included the resistance of the rectifier bridge circuit in both the cases.
- (iii) There may be variation in the temperature due to which the electrical resistance of the bridge changes.
- (iv) Bridge rectifier has imperfect capacitor & diodes. due to this it bypasses the high frequency elements. Hence, there is decrement in reading.
- (v) The sensitivity of rectifier type instrument is low in case of A.C input voltage.

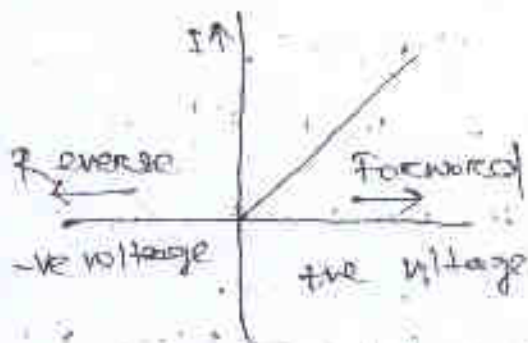
* Rectifier type Instrument →

- Rectifier instruments are used for both A.C & D.C measurement by using a rectifier circuit to convert A.C or D.C into an unidirectional D.C and then use a D.C meter to indicate the value of rectified D.C.
- PMMC instrument is used as D.C meter bcoz it has higher sensitivity than other electrodynamic meters type or M.I. type instrument.

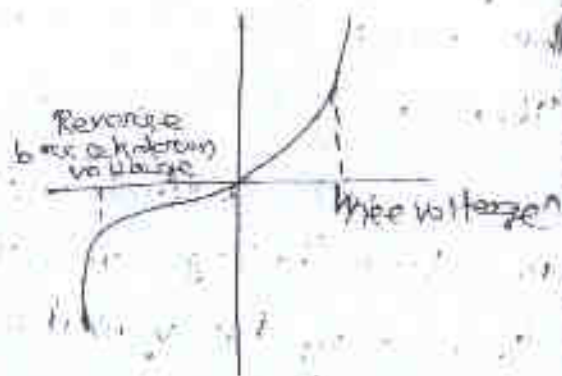
Rectifier characteristics

Characteristics: An ideal rectifying element would be current voltage relationship such as it is linear in the forward direction.

Practical char.

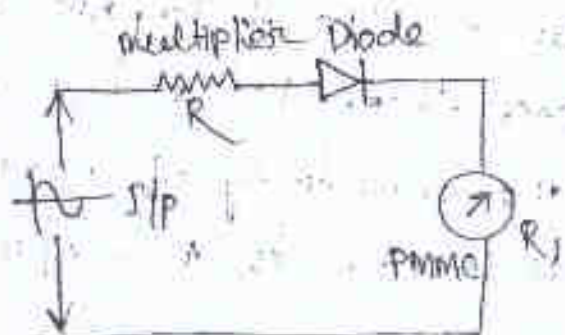


(Ideal characteristic)



Half wave Rectifier circuit of Rectifier type instruments \rightarrow

Let's consider a circuit given below in which the rectifying element is connected in series with sinusoidal voltage permanent magnet moving coil (PMMC) instrument and the multiplier resistance.



\rightarrow It is very essential to limit the current drawn by the [PMMC] permanent magnet moving coil instrument because if the current exceeds the current rating of PMMC then it destroys the instrument. So we have connected a multiplier resistance.

Now, full scale deflection deflection

$$\text{Current, } I = \frac{V}{R + R_1} \text{ (for D.C. case)}$$

$V =$ R.M.S value of voltage.

$R =$ Multiplier resistance

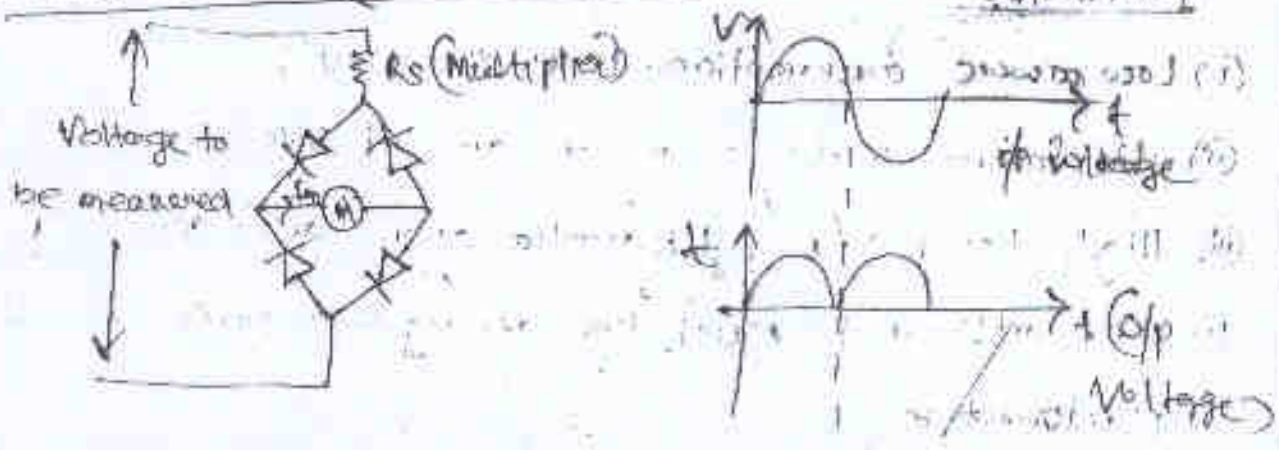
R_i = Resistance of PMMC instrument. (19)
 And for A.C i/p $V_{avg} = \frac{V_m}{\pi} = \frac{V_{rms}}{\sqrt{2}}$ [V_m = max value]

\therefore form factor = $\frac{V_{rms}}{V_{avg}} = \frac{V_{rms}}{\frac{V_m}{\pi}} = \frac{V_{rms}}{\frac{V_{rms}}{\sqrt{2}} \cdot \pi} = \frac{\sqrt{2}}{\pi} \approx 2.22$

Effect of wave form

From the above data it is assumed that the instrument must be marked in terms of 2.22 times the current actually measured. A no. of current wave forms having the same avg. value may have Rms value which may vary considerably.

Full wave Rectifier circuit



form factor = $\frac{\text{Rms value of a.c. wave}}{\text{Avg. value of a.c. wave}}$
 $= \frac{V_{rms}}{V_{avg}} = \frac{V_{rms}}{\frac{V_m}{\pi}} = \frac{V_{rms} \cdot \pi}{V_m} = \frac{V_{rms} \cdot \pi}{\sqrt{2} V_{rms}} = \frac{\pi}{\sqrt{2}} \approx 2.22$

(Controlling torque) - Controlling torque is provided by control spring, is a hair spring made of phosphor bronze. (20)

$$T_c \propto \theta \quad (\theta = \text{deflection Angle})$$

At equilibrium $T_d = T_c$

$$\therefore \theta \propto I$$

Since the deflection is directly proportional to current, such instruments have uniform scale.

* Damping torque - It is provided by eddy current damping.

ADVANTAGES

- (i) Low power consumption. (25-200mW)
- (ii) Uniform scale over an arc of 270°
- (iii) High torque/weight ratio shows good accuracy.
- (iv) Not affected much by stray magnetic field.

Disadvantages

- (i) Costly as compared to M.I. instrument.
- (ii) Errors arise due to ageing of control spring.
- (iii) Its use is limited to D.C. only.

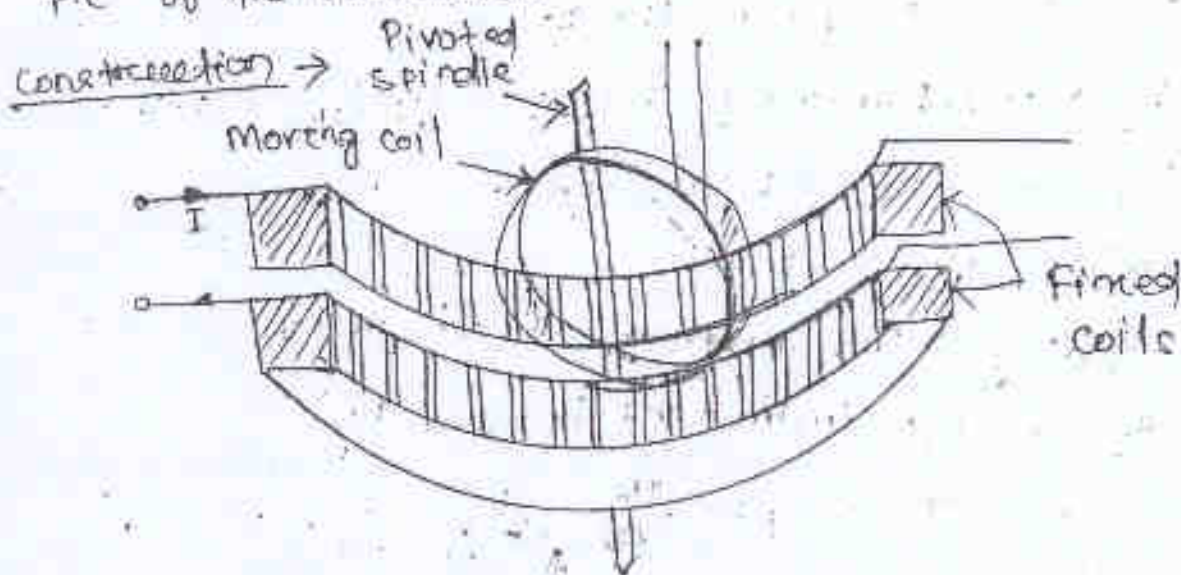
Errors

- (i) frictional error. Due to friction of moving parts.
- (ii) Effects due to ageing of permanent magnet and control spring.
- (iii) Temp. error.

Dynamometer Type Watt meters

(2/2)

These instruments are similar in design & construction to electrodynamometer type wattmeters and voltmeters. When the instrument is used as wattmeter, the fixed coils are connected in series with the load and hence carry the current in the circuit. The fixed coils therefore form the current coil or simply C.C. of the wattmeter. The moving coil is connected across the voltage and hence carries a current proportional to the voltage. A high non-inductive resistance is connected in series with the moving coil to limit the current to a small value. The moving coil is called the pressure coil or voltage coil or simply P.C. of the wattmeter.



(i) Fixed coils → Fixed coils which carry the current in the circuit are divided into two halves. They are wound with heavy wire. The wire is laminated or stranded in order to avoid eddy current losses in conductors especially when carrying heavy currents.

The maximum current range of wattmeter is about 20A.

(ii) Moving coil → Moving coil is mounted on a pivoted spindle and is entirely embraced by the fixed coils.

The spring control is used to control the movement. The current of the moving coil is carried by instrument springs, it is limited to the values which can be carried safely by springs without appreciable heating. A resistance is connected in series with the voltage circuit, in which current is limited up to 100 mA. The voltage rating of the wattmeter is limited to about 600V.

Control → Spring control is used to control the instrument.

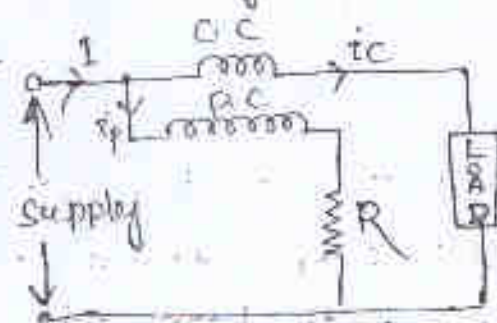
Damping:- The friction damping is used. Electromagnetic or eddy current damping is not used as introduction of permanent magnet distorts the weak operating magnetic field.

Scale and Pointers → They are equipped with microscope type scales and knife edge pointers to remove reading errors due to parallax.

Theory:- The instantaneous torque of an electro-dynamometer instrument is given by

$$T_i = t_p t_c \frac{dM}{d\theta}$$

where t_p, t_c are the instantaneous values of current in two coils.



Let V and I be the rms values of voltage and current being measured.

Instantaneous value of voltage across the pressure coil, $V_i = V \sin \omega t$.

Here the pressure coil ckt is highly resistive, i_p with the wattmeter.

$$E_p = \frac{V \sin \phi}{R_p} \approx \frac{V \sin \phi}{R_p} \sin \omega t$$

where I_p is the rms value of the current in the coil

R_p = resistance of pressure coil circuit

If the current through the current coil lags the voltage by an angle ϕ

$$i_c = \sqrt{2} I \sin(\omega t - \phi)$$

∴ Instantaneous torque

$$T_i = \sqrt{2} I_p \sin \omega t \times \sqrt{2} I \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$= 2 I_p I \sin \omega t \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$= I_p I [\cos \phi - \cos(2\omega t - \phi)] \frac{dM}{d\theta}$$

∴ Average deflecting torque

$$T_{av} = \frac{1}{T} \int_0^T T_i d(\omega t)$$

$$= \frac{1}{T} \int_0^T I_p I [\cos \phi - \cos(2\omega t - \phi)] \frac{dM}{d\theta} \cdot d(\omega t)$$

$$= I_p I \cos \phi \cdot \frac{dM}{d\theta}$$

$$= \left(\frac{V I}{R_p} \right) \cos \phi \frac{dM}{d\theta}$$

Controlling torque $T_c = K \theta$

where K = spring constant

θ = final steady deflection

Now, at steady position,

$$K \theta = I_p I \cos \phi \cdot \left(\frac{dM}{d\theta} \right) / k$$

$$= \left(\frac{V I \cos \phi}{R_p k} \right) \frac{dM}{d\theta} = K_1 V I \cos \phi \cdot \frac{dM}{d\theta}$$

$$= \left(K_1 \frac{dM}{d\theta} \right) \cos \phi \quad \text{--- (1)}$$

where ϕ = power factor measured by wattmeter

$$= V I \cos \phi \text{ and } K_1 = \frac{1}{R_p k}$$

Induction type wattmeter type wattmeter :-

(2A)

1) Error due to pressure coil Inductance \rightarrow

Let, r_p = resistance of pressure coil

L = Inductance of pressure coil

R = Resistance in series with pressure coil

R_p = Total resistance of pressure coil $\text{ckt} = r_p + R$

V = Applied voltage to pressure coil ckt

I = Current in the current coil ckt

I_p = Current in pressure coil ckt

Z_p = Impedance of pressure coil ckt

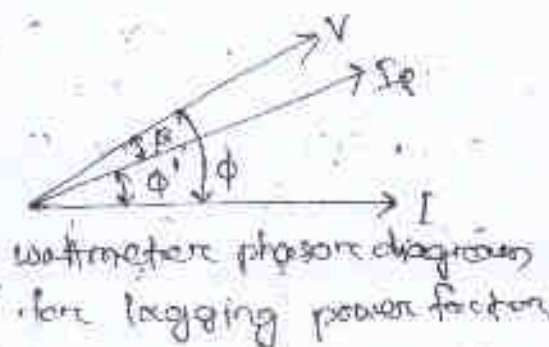
$$= \sqrt{(R + r_p)^2 + (\omega L)^2}$$

β = angle by which current in the pressure coil lags the voltage.

$$= \tan^{-1} \left(\frac{\omega L}{R_p} \right) = \tan^{-1} \left(\frac{\omega L}{r_p + R} \right)$$

ϕ = angle betⁿ applied voltage and load current

From the phase diagram, the angle betⁿ the p.c. current and c.c. current is $\phi' = \phi - \beta$



\therefore The actual wattmeter

$$\text{reading is } \left(\frac{I_p I}{K} \right) \cos \phi' \cdot \frac{dM}{d\theta} = \left(\frac{V}{Z_{pk}} \right) I \cos(\phi - \beta) \cdot \frac{dM}{d\theta}$$

where $Z_p = \frac{R_p}{\cos \beta}$

Actual watt meter reading -

$$\frac{VI \cos(\phi - \beta)}{K R_p} \cdot \frac{dM}{d\theta} = \frac{VI}{K R_p} \cos(\phi - \beta) \cdot \frac{dM}{d\theta}$$

In the absence of inductance $Z_p = R_p$ and $\phi = 0$ (25)
 Therefore the wattmeter will read true power under these condition.

$$\therefore \text{True power} = \frac{|I_p|}{K} \cos \phi \frac{dM}{d\theta} = \frac{VI \cos \phi}{KR_p} \frac{dM}{d\theta} \quad \text{--- (iii)}$$

Now, $\frac{\text{True power}}{\text{Actual wattmeter reading}}$

$$= \frac{\frac{VI \cos \phi}{KR_p} \frac{dM}{d\theta}}{\frac{VI \cos(\phi - \beta)}{KR_p} \cos \beta \frac{dM}{d\theta}} = \frac{\cos \phi}{\cos \beta \cos(\phi - \beta)} \quad \text{--- (iv)}$$

$$\therefore \text{True power} = \frac{\cos \phi}{\cos \beta \cos(\phi - \beta)} \times \text{actual wattmeter reading}$$

The correction factor is a factor by which a actual wattmeter reading multiplied to get true power.

$$\therefore \text{Correction factor} = \frac{\cos \phi}{\cos \beta \cos(\phi - \beta)} \text{ for lag. loads}$$

$$\text{Similarly correction factor} = \frac{\cos \phi}{\cos \beta \cos(\phi + \beta)} \text{ (for leading loads)}$$

In case of lagging loads, the error in terms of instrument deflection is

Actual wattmeter reading, - True power

$$= \left[1 - \frac{\cos \phi}{\cos \beta \cos(\phi - \beta)} \right] \times \text{actual wattmeter reading}$$

$$= \left[1 - \frac{\cos \phi}{\cos(\phi - \beta)} \right] \times \text{actual wattmeter reading}$$

As β is very small, $\cos \beta$ is nearly unity

$$\left[\frac{\cos \phi}{\cos \phi \cos \beta + \sin \phi \sin \beta} \right] \times \text{actual wattmeter reading}$$

$$= \left[\frac{\sin \phi \sin \beta + \cos \phi \cos \beta - \cos \phi}{\cos \phi \cos \beta + \sin \phi \sin \beta} \right] \times \text{actual wattmeter reading}$$

$$= \left[\frac{\sin \phi \sin \beta}{\cos \phi + \sin \phi \cdot \sin \beta} \right] \times \text{actual wattmeter reading} \quad [\because \cos \beta \approx 1]$$

$$= \left[\frac{\sin \beta}{\cot \phi + \sin \beta} \right] \times \text{actual wattmeter reading}$$

Now, eqⁿ (iv) can be written as

$$\frac{\text{True power}}{\text{actual wattmeter reading}} = \frac{\cos \phi}{\cos \beta \cos (\phi - \beta)}$$

$$= \frac{\cos \phi}{\cos^2 \beta \cos \phi + \sin \phi \sin \beta \cdot \cos \beta} = \frac{\cos \phi}{\cos^2 \beta (\cos \phi + \sin \phi \cdot \tan \beta)}$$

$$= \frac{\frac{1}{\cos^2 \beta}}{\frac{\cos \phi}{\cos \phi} + \tan \beta \cdot \frac{\sin \phi}{\cos \phi}} = \frac{\sec^2 \beta}{1 + \tan \phi \cdot \tan \beta}$$

$$= \frac{1 + \tan^2 \beta}{1 + \tan \phi \cdot \tan \beta}$$

Now β is very small, hence $\tan^2 \beta \ll 1$.

$$\frac{\text{true power}}{\text{actual wattmeter reading}} = \frac{1}{1 + \tan \phi \cdot \tan \beta}$$

Actual wattmeter reading = true power $(1 + \tan \phi \tan \beta)$

$$\begin{aligned} \text{Error} &= \text{Actual wattmeter reading} - \text{true power} \\ &= \tan \phi \tan \beta \times \text{true power} \end{aligned}$$

Percentage error = $\frac{\text{Actual Power} - \text{True Power}}{\text{True Power}} \times 100$
 $= \tan \phi \cdot \tan \beta \times 100$

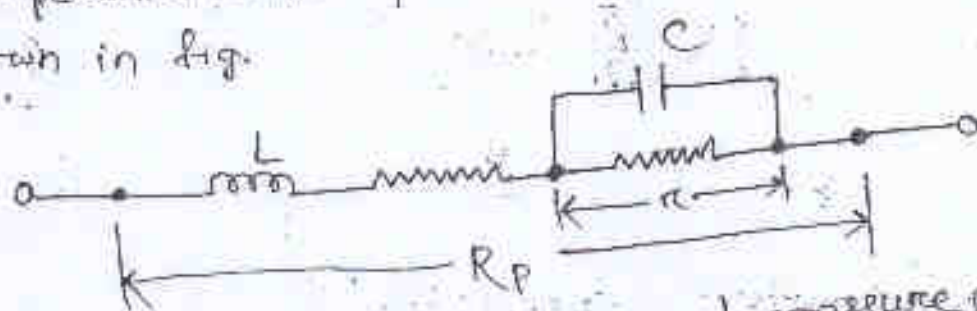
But true power = $VI \cos \phi$

Error = $\tan \phi \cdot \tan \beta \times VI \cos \phi = VI \sin \phi \tan \beta$

Hence eqⁿ (VII) shows that error is very serious at low power factor.

* Compensation for Inductance of pressure coil

The errors caused by the inductance of pressure coil can be compensated by connecting a capacitor in parallel with a portion of a series resistance as shown in fig.



(Compensation for inductance of pressure coil ok)

Now total impedance of the circuit

$$Z_p = (R_p - \pi) + j\omega L + \frac{\pi - j\omega C \pi^2}{1 + \omega^2 C^2 \pi^2}$$

If the value of circuit constant is so chosen, that for power frequencies $\omega^2 C^2 \pi^2 \ll 1$

$$Z_p = R_p - \pi + j\omega L + \pi - j\omega C \pi^2 \approx R_p + j\omega(L - C\pi^2)$$

If we make $L = C\pi^2$, $Z_p = R_p$ and $\beta = 0$

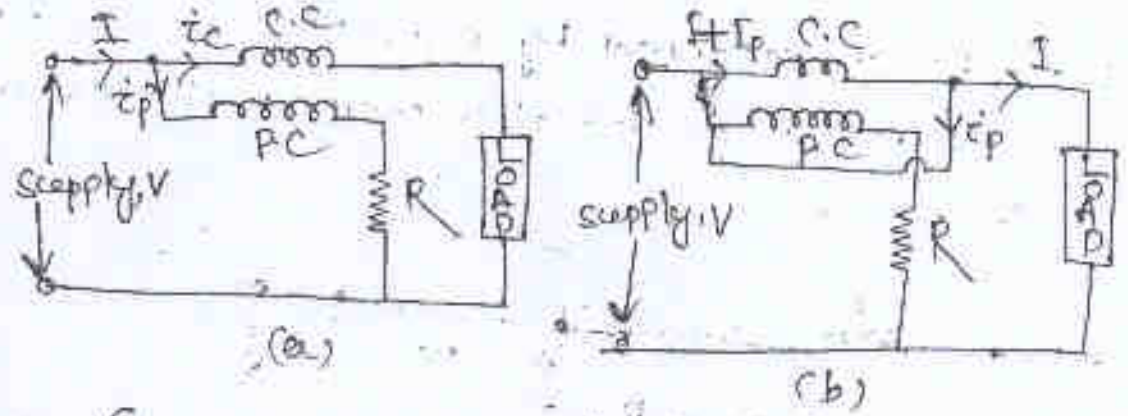
Hence error caused by pressure coil inductance is completely eliminated.

Pressure coil capacitance.

The pressure coil circuit may possess capacitance mainly due to interturn capacitance of the series resistance. In the absence of inductance, capacitance of pressure coil will introduce error. But generally both inductance and capacitance are present and cancelling the effect of each other.

Errors due to Connections

There are two methods of connecting a wattmeter in the circuit as shown in fig. below.



(Wattmeter connection)

In fig. (a),

Power indicated by wattmeter = Power consumed by load + power lost in current coil.

In fig. (b)

Power indicated by wattmeter = Power consumed by load + power lost in pressure coil.

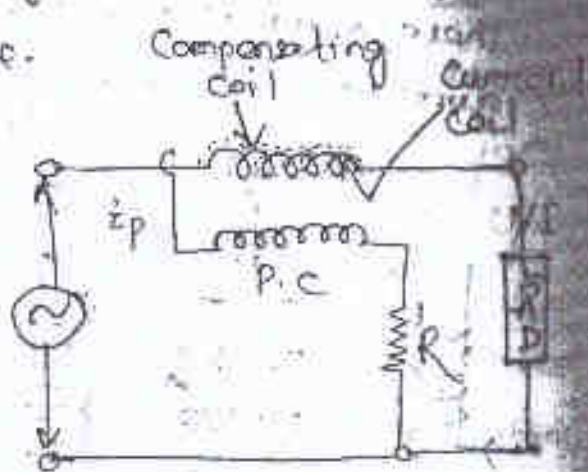
Hence neither connection measures the power in the load directly, without correction.

In fig. (a), the voltage coil is connected on the supply side of current coil, hence voltage applied to the voltage coil is higher than that of the load and the voltage drop in the current coil.

By using other current coils taken by the voltage coil and the

If the load current is small, the drop in the current is small, so the method of connection introduces a very small error. On the other hand, if the load current is large, the power lost in the voltage coil will be small compared with the power in the voltage and the secondary of connection is preferable.

To overcome the error because of coil carrying the pressure coil current in addition to load current, the wattmeters are provided with compensating winding, connected in series with the potential coil but placed so that, it produces a field in the opposite direction to that of current coil. Thus if no load current flows in the instrument, the deflection should be zero.



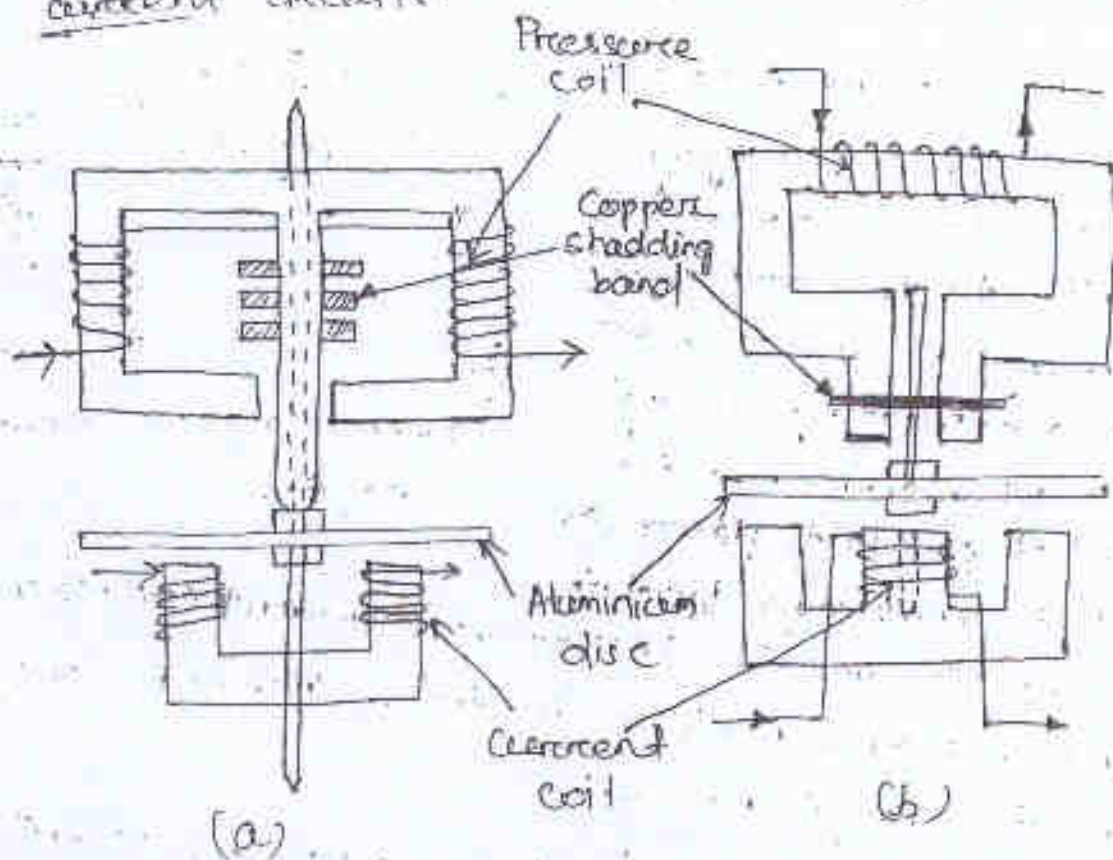
(Compensating coil connection diagram)

Eddy current Error: The alternating magnetic field of current coil induces eddy currents in the solid metal parts nearby the current coil, hence alter the magnitude and phase of the magnetic field. ^(ind) Such error is produced. This can be eliminated by removing the metal parts from the current coil. If the coil is designed for heavy currents, it should consist of stranded conductors in order to minimize the flow of eddy current through the conductors.

As in the above circuit, the other effects like temperature, stray magnetic field, friction, heating and vibration of moving system may introduce errors in the wattmeter.

Induction Type Wattmeter

The working principle of induction wattmeter is same as that of the induction ammeter & voltmeter. These wattmeters, can only be used on alternating current circuits.



(Induction type wattmeter)

Construction:- The wattmeters have two laminated electromagnets: series magnet, and shunt magnet. Series magnet is excited by the load current and shunt magnet is excited by a current proportional to voltage across the load. The coils of series and shunt magnets are called current coil and pressure coil respectively. The moving system consist of a thin aluminium...

So, connected that is cuts the fluxes of both the shunt and series magnets. Two torques act on the disc, one is produced by the interaction between the flux of series magnet and eddy current induced in the disc by the flux of shunt magnet and other by the interaction between the flux of series magnet and the eddy current induced in the disc by the flux of series magnet. The resultant torque is proportional to the power of the load. One or more adjustable shading bands of copper are fitted on one limb of the shunt magnet in order to cause its flux to lag in phase by exactly 90° behind the applied voltage.

In fig (a), the two pressure coils connected in series are wound so that both of them send flux through the central limb. The two current coils, connected in series are wound so that they both magnetize the common core in the same direction. The copper shading bands are fitted on the central limb.

In fig. (b) shows the wattmeter which consists of only one pressure coil and one current coil. An adjustable copper shading band surrounds the two projecting pole pieces of the shunt magnet and is used to adjust the phase of the flux of this magnet.

Controlling system is provided by springs & have eddy current damping. The scale is long and uniform (up to 300°). The current range is about 100A.

The net deflecting torque acting on the disc, on
 account of the fluxes of the shunt and series magne-
 tising is given by

$$T_d \propto \frac{\phi_{sh} \phi_{se} f \cos \alpha \sin \beta}{Z} \quad \text{--- (17)}$$

where ϕ_{sh} = rms value of the shunt flux

ϕ_{se} = rms value of the series flux

f = supply frequency

Z = impedance of the eddy current path

α = angle by which eddy current lags behind
 the eddy emf

β = phase angle betⁿ ϕ_{sh} and ϕ_{se}

Here ϕ_{sh} is made to lag by nearly 90° behind the
 applied voltage and ϕ_{se} is in phase with I . The angle
 by which I lags behind the applied voltage is ϕ .

Hence the phase angle between the ϕ_{sh} and ϕ_{se} is
 given by

$$\beta = 90^\circ - \phi$$

The quantities Z and α are frequency dependent
 but their variation is small and hence may be treated
 as constant.

$$\text{Also } \phi_{se} \propto I, \quad \phi_{sh} \propto \frac{V}{f}$$

$$\text{and } \cos \alpha \approx 1 \quad (\text{as } \alpha \text{ is very small})$$

Therefore eqⁿ (17) may be simplified as

$$T_d \propto VI \sin(90^\circ - \phi)$$

$$\propto VI \cos \phi$$

Advantages and disadvantages of Induction wattmeter

Advantages

- (i) Since the operating fluxes are strong, the effects of stray magnetic fields are negligible.
- (ii) Robust in construction and scale is as long as 300.

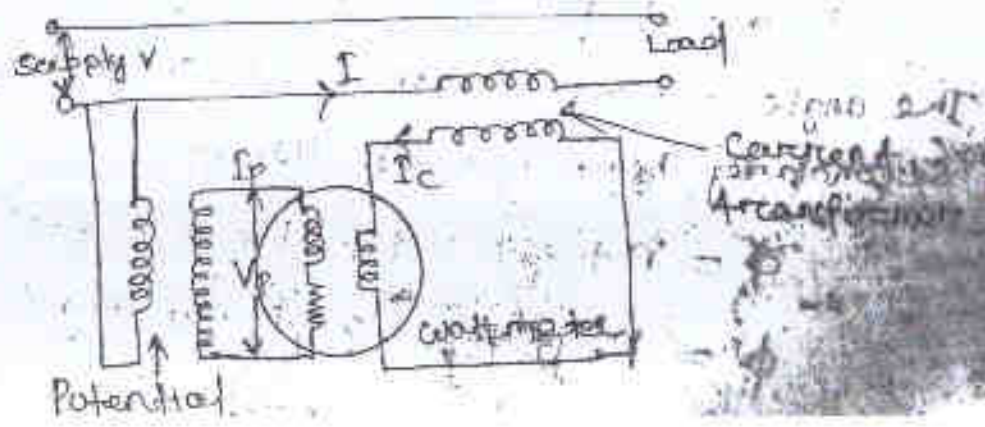
Disadvantages

- (i) Operating torque is dependent of frequency.
- (ii) Power consumption is high.
- (iii) Have a heavy moving system.
- (iv) Have first grade accuracy of only at a stated frequency and temperature.
- (v) Can only be used on a.c. circuits.

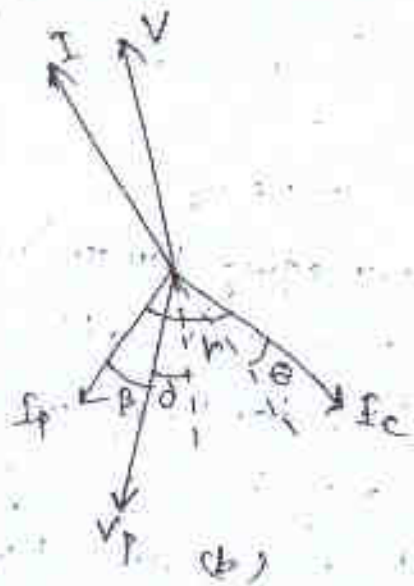
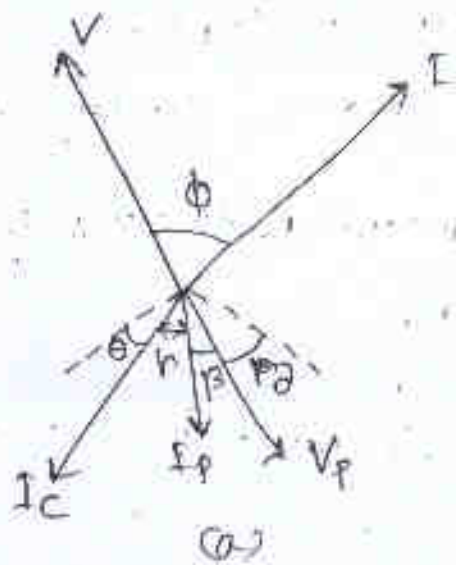
* Extension of Wattmeter Range →

Extension of wattmeter range means extension of range of current coil, extension of range of p.c or extension of both. Instrument transformers are used for this purpose.

Current and voltage transformers of different ratios are used to supply the wattmeter current and p.c respectively. The connections of a wattmeter used with instrument transformers are shown in fig. As current and voltage transformers introduce ratio and phase angle errors, corrections must be applied to allow for these errors.



From the diagram, we can see the current and voltage of the load and the wattmeter are shown in fig. (a) lagging and leading power factor.



Let, V = Voltage of the load

I = load current

ϕ = load power factor angle

V_p = Voltage applied to the P.C of wattmeter

I_c = Current through the C.C of wattmeter

I_p = Current through pressure coil

η = Phase angle between I_c and V_p

β = angle by which I_p lags V_p

δ = phase angle of voltage transformer

θ = phase angle of current transformer

From vector diagram

$$\phi = \eta + \theta + \delta + \beta \quad [\text{From fig (a)}]$$

$$\phi = \eta - \theta - \delta - \beta \quad [\text{From fig (b)}]$$

The angle δ may be +ve or -ve depending on the secondary bars position. Hence

$$\phi = \eta + \theta \pm \delta + \beta \quad [\text{From fig (a)}]$$

$$\phi = \eta - \theta \pm \delta - \beta \quad [\text{From fig (b)}]$$

There are three kind of corrections to be made in the wattmeter reading. They are:

- (i) Correction applied for I_p lagging V_p due to the pressure coil inductance.
- (ii) Correction for ratio error in the C.T. Transformer.
- (iii) Correction for ratio error in the P.T. Transformer.

The correction factors of the wattmeter are:

$$K = \frac{\cos \phi}{\cos \phi \cos (\phi - \theta \pm \delta - \beta)} \quad [\text{in fig (a)}]$$

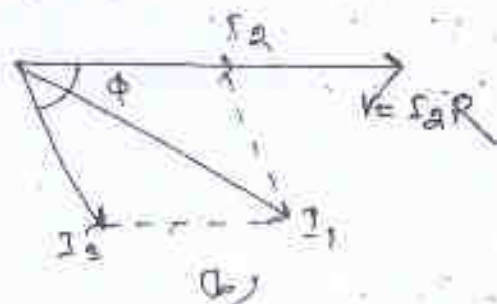
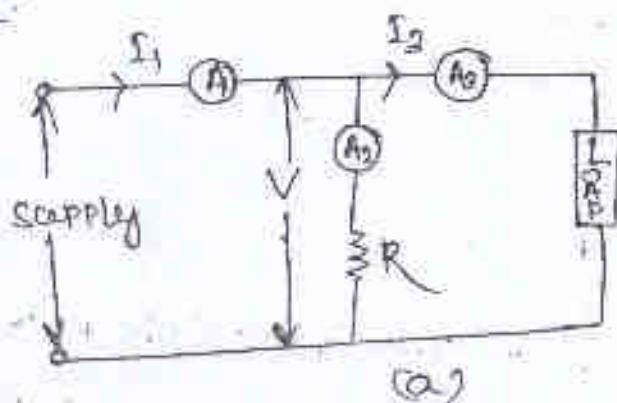
$$K = \frac{\cos \phi}{\cos \phi \cos (\phi + \theta \pm \delta + \beta)} \quad [\text{in fig (b)}]$$

Therefore correct reading of the wattmeter will be $P = k \times \text{wattmeter reading} \times \text{Actual ratio}$

Measured power in of C.T \times Actual ratio of P.T

Three Ammeter Method:-

Ammeter A_3 measure the load current. A_2 the current through the non-inductive resistance connected across the load and A_1 the vector sum of current measured by A_3 and A_2 . From the vector diagram,



$$I_1^2 = I_2^2 + I_3^2 + 2I_2I_3 \cos \phi$$

Let $I_2 R = V$

$$\therefore I_1^2 = I_2^2 + I_3^2 + \frac{2VI_3}{R} \cos \phi$$

$$\text{or } VI_3 \cos \phi = \frac{(I_1^2 - I_2^2 - I_3^2)R}{2}$$

which is the power in the load.

the difference in operation of current transformer (C.T) & potential transformer

Current Transformer Potential transformer

- | | |
|---|--|
| (i) Secondary must always be shorted. | (i) Secondary is nearly under open ckt condition. |
| (ii) The winding carries full line current. | (ii) The winding is impressed with full line voltage. |
| (iii) The primary current is independent of the secondary ckt conditions. | (iii) The primary current depends on secondary circuit conditions. |
| (iv) It can be treated as series transformer under short ckt conditions. | (iv) It can be treated as parallel transformer under open ckt secondary. |
| (v) A small voltage exists across it's terminal as connected in series. | (v) Full line voltage appears across it's terminals. |

* Instrument transformer - The transformer used in conjunction with measuring instrument for measurement purpose is called "Instrument transformer."

Types of torque developed in the instrument

For proper functioning of instrument the following forces are employed.

① Deflecting Torque:- The force or torque which brings the needle (pointer) in motion is called deflecting torque or deflecting force. This force is obtained by magnetic effect or heating effect of electricity.

② Controlling Torque:- The force or torque which controls the movement of pointer caused by deflecting and brings the pointer at its original position when there is no deflecting force, is called controlling torque.

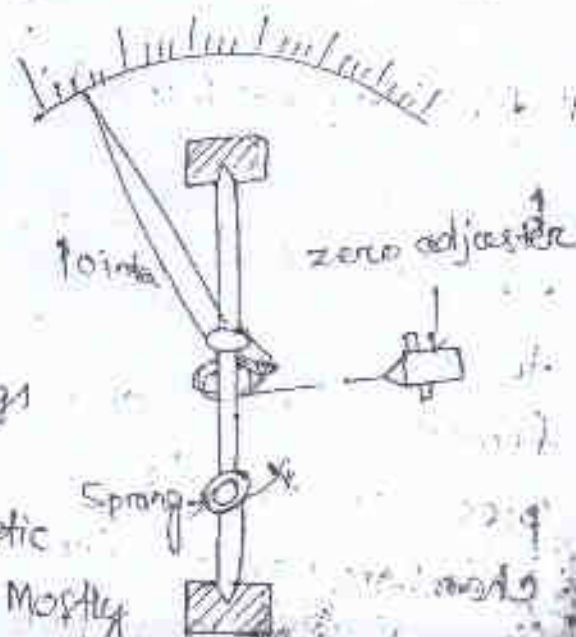
There are two types of producing controlling force:

- (a) Spring control
- (b) Gravity control

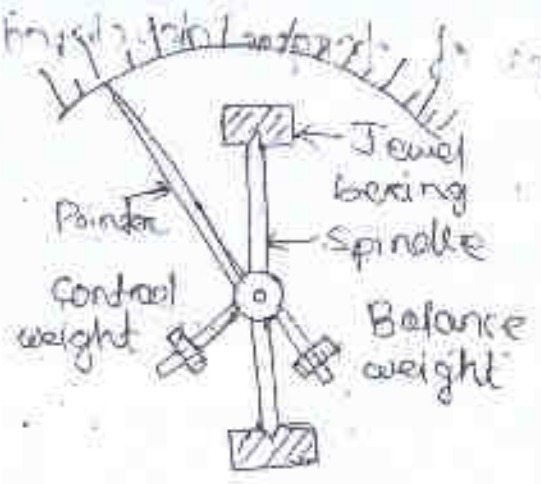
(a) Spring control:-

In this type of controlling force a special type of spring made of phosphor bronze are used. These springs have properties of low specific resistance, non-magnetic

& low temp co-efficient etc. Mostly two springs attached with lower & upper position of the spindle are used. Its function is to provide controlling force.



Controlling Condition :- In this type of controlling force a small calculated weight is attached to one end of the spindle which works on the principle of gravitational force, so it is called Gravity control. The direction of controlling force is always opposite to deflecting force.

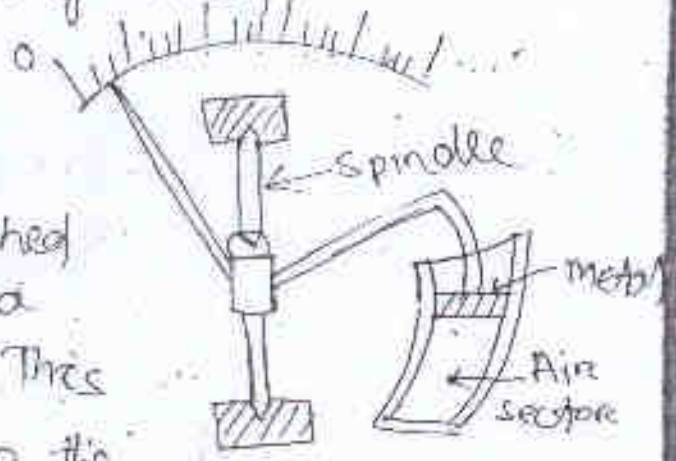


* Damping Torque :- The force or torque which brings the pointer in its original position very quickly e.g. minimize the oscillating effect (vibration effect) is called damping force or torque. There are three methods to obtain this force.

- (i) Air damping method
- (ii) Eddy current damping method
- (iii) Fluid friction damping method

(i) Air Damping Method :-

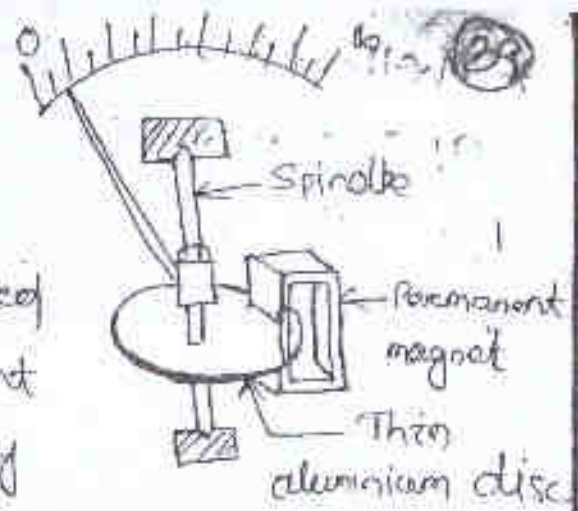
A small-light weight piece of aluminium attached to pointer is placed in a fixed closed chamber. This piece of metal moves in the chamber as pointer deflects and



due to air resistance, it brings the needle in its proper position. The damping is produced due to friction of air, so it is called air damping.

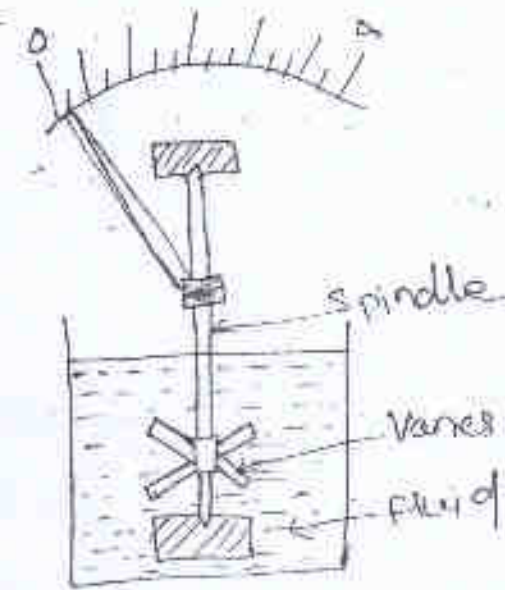
(b) Eddy current damping :-

A small-light weight disc normally made of aluminium attached to spindle is rotated in between poles of permanent magnet and produces eddy current in disc. We know, as per Lenz's law, the direction of an induced eddy current force is always opposite to the cause. This opposition acts as an eddy current damping force.



(c) Fluid Friction Damping :-

In this type of system, in place of air some liquid is used in which small metal piece or disc attached to one end of the spindle is placed in fluid. Due to friction of fluid, a force is produced which minimizes the vibration effect. This force is called fluid friction damping force.



Methods to extend the range of voltmeter and ammeter:-

Ans.- The extension of range of voltmeters and meters are possible by using the following method

Here, R_m = meter resistance

R_{sh} = shunt resistance

I_{sh} = shunt current

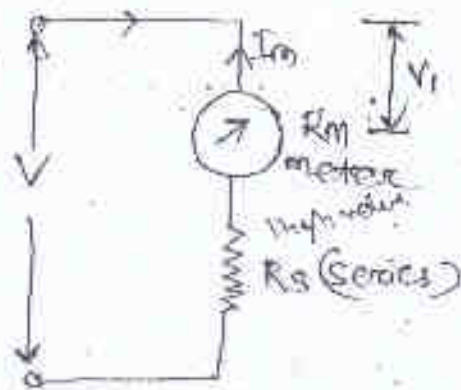
I_m = meter current

V_m = voltage across the meter

Shunt connection (Ammeter):-

VOLTMETER
Series multiplier:-

Let us consider a voltmeter is connected in series with a resistor (R_s) across a source voltage V .



$$V = V_m + I_m R_s$$

$$\Rightarrow I_m R_s = (V - V_m)$$

$$\Rightarrow R_s = \frac{V - V_m}{I_m} = \frac{V}{I_m} - \frac{V_m}{I_m} = \frac{V_m}{I_m} - R_m$$

($\because \frac{V_m}{I_m} = R_m$)

Multiplying factor (m) = $\frac{V}{V_m}$

$$\text{i.e. } m = \frac{V_m + I_m R_s}{V_m} = \frac{I_m (R_m + R_s)}{I_m R_m}$$

$$\Rightarrow m = 1 + \frac{R_s}{R_m}$$

$$\Rightarrow \frac{R_s}{R_m} = m - 1$$

$$\text{OR } \boxed{R_s = R_m (m - 1)}$$

As m is large quantity

$$\therefore R_s = R_m \times (1000 - 1) = 999 R_m$$

which implies that series resistance be of high value. Hence in order to extend range of a voltmeter a series resistance value is to be added.

Ammeter (shunt multiplier) :-

In this technique a shunt resistance (R_{sh}) will be added across the metre.

$$\therefore V = I_m R_m = I_{sh} \cdot R_{sh}$$

$$\Rightarrow R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{I_m R_m}{I - I_m}$$

$$\Rightarrow \frac{I - I_m}{I_m} = \frac{R_m}{R_{sh}}$$

$$\Rightarrow \frac{I}{I_m} - 1 = \frac{R_m}{R_{sh}}$$

$$\Rightarrow \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}} \Rightarrow \frac{R_m}{R_{sh}} = \frac{I}{I_m} - 1 = (m - 1)$$

$$\Rightarrow \boxed{R_{sh} = \frac{R_m}{m - 1}}$$

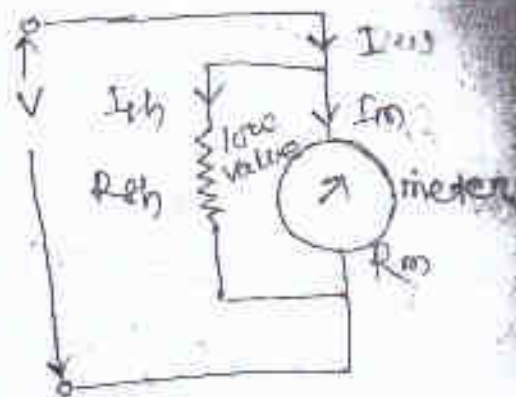
where $m = \frac{I}{I_m}$ = multiplying factor

The multiplying factor (m) is high value

$\Rightarrow (m - 1)$ is also high.

$\Rightarrow \frac{1}{m - 1}$ will be very low. $\Rightarrow R_{sh}$ will very low.

Hence in order to extend the range of an ammeter, a shunt resistance (R_{sh}) of very low value will be added in parallel to the meter.



The torque of an ammeter varies as the square
 of the current through it. If a current of 10A,
 produces a deflection of 90° , what deflection will
 occur for a current of 5, when the instrument
 is (i) Spring control
 (ii) Gravity control

Ans:- Since deflecting torque varies as (current)²

we have $T_d \propto I^2$

for spring control: $T_d \propto \theta$

$\therefore \theta \propto I^2$

for gravity control: $T_d \propto \sin \theta$

$\therefore \sin \theta \propto I^2$

(i) For spring control

$90^\circ \propto 10^2$ and $\theta \propto 5^2$

$$\theta = 90^\circ \times \frac{5^2}{10^2} = 22.5^\circ$$

(ii) For gravity control

$\sin 90^\circ \propto 10^2$ and $\sin \theta \propto 5^2$

$$\frac{\sin \theta}{\sin 90^\circ} = \frac{5^2}{10^2} = \frac{25}{100}$$

$$\Rightarrow \sin \theta = \frac{1}{4}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{4}\right) = 14.47^\circ$$

Electrodynamometer

Advantages:

- ① As the coils are coaxial, these instruments are free from hysteresis and eddy current errors.
- ② They have a precision grade accuracy.
- ③ These instruments can be used both A.C + D.C.
- ④ This type's of voltmeters are very useful where accurate rms values of voltage, irrespective of waveforms are reqd.

Disadvantages:

- (i) They have a low torque/weight ratio and hence have a low sensitivity.
- (ii) Low torque/weight ratio gives increased frictional losses.
- (iii) They are more expensive than either the PMMC or the M.I. type instrument.
- (iv) These instrument are sensitive to overloads and mech. impacts.
- (v) The operating current of these instrument is large ~~owing to the~~
- (vi) They have non-uniform scale.

Measurement of inductance

(i) Maxwell's Bridge method

(ii) Owen Bridge method

* Measurement of capacitance

(i) De-Sauty Bridge method

(ii) Schering Bridge method

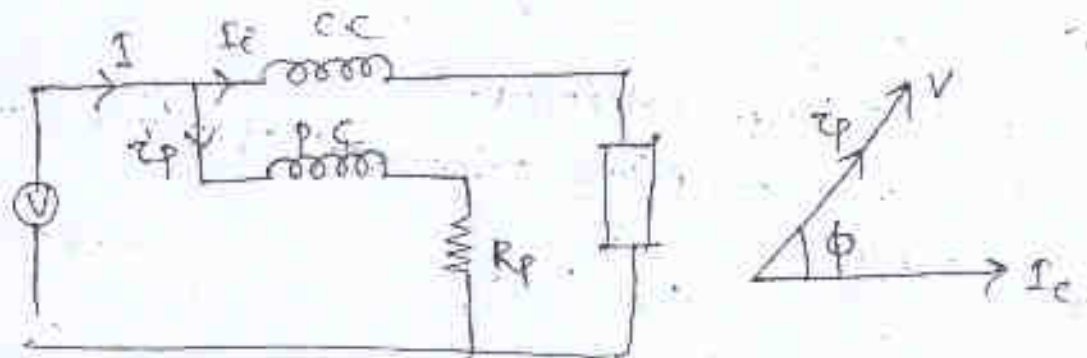
(iii) LCR Bridge method

* Measurement of resistance

(i) Low \rightarrow voltage drop & potentiometer

(ii) medium \rightarrow wheat stone bridge & substitution

(iii) High - loss of charge method



$$v_p = v_{p \max} \sin \omega t$$

$$= \sqrt{2} \times I_p \sin \omega t$$

$$i_c = i_{c \max} \sin(\omega t - \phi)$$

$$= \sqrt{2} \times I_c \sin(\omega t - \phi)$$

$$\tau_d = \tau_p \tau_c \frac{dM}{d\theta}$$

$$\tau_d = \sqrt{2} \times I_p \sin \omega t \times \sqrt{2} \times I_c \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$\int_0^{2\pi} \tau_d = \frac{2 \times 2 \times I_p \times I_c \times \frac{dM}{d\theta}}{2\pi} \int_0^{2\pi} \sin \omega t \cdot \sin(\omega t - \phi) dt$$

$$\Rightarrow \tau_d = I_p I_c \frac{dM}{d\theta} \cdot \frac{1}{2\pi} \int_0^{2\pi} [\cos \phi - \cos(\omega t - \phi)] dt$$

$$= I_p I_c \frac{dm}{dt} \times \frac{1}{2\pi} \cos \phi \int_0^{2\pi} \cos \theta \times d\theta$$

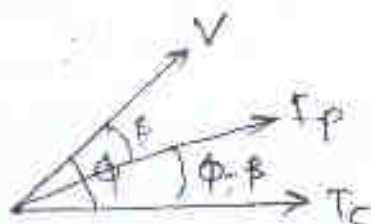
$$T_d = I_p I_c \cos \phi \frac{dm}{dt}$$

$$= \frac{V}{R_p} I_c \cos \phi \frac{dm}{dt}$$

$$T_d = \frac{P}{R_p} \frac{dm}{dt}$$

$$T_c \propto \theta$$

$$T_d = T_c \quad \theta \propto P$$



$$\cos \beta = \frac{R_p}{Z_p}$$

$$\Rightarrow Z_p = \frac{R_p}{\cos \beta}$$

Actual

$$T_d = I_p I_c \cos(\phi - \beta) \frac{dm}{dt}$$

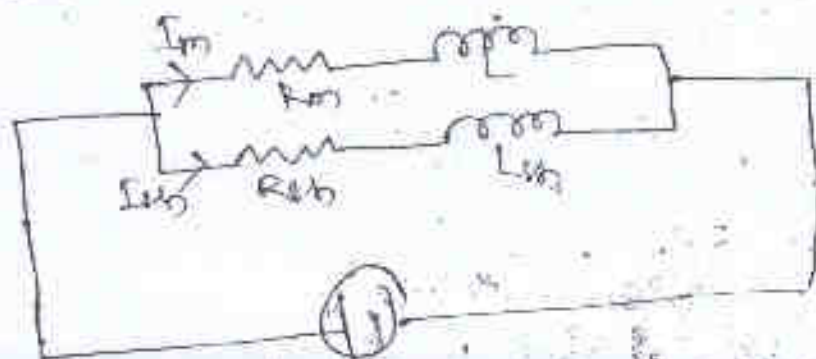
$$= \frac{V}{Z_p} I_c \cos(\phi - \beta) \frac{dm}{dt}$$

$$= \frac{V}{R_p} I_c \cos(\phi - \beta) \cdot \cos \beta \cdot \frac{dm}{dt}$$

$$R_{ch} = \frac{R_m}{m-1}$$

M.I

$$\frac{I_{ch}}{I_m} = \frac{R_m}{R_{ch}} \sqrt{\frac{1 + \omega^2 \left(\frac{L_{ch}}{R_m}\right)^2}{1 + \omega^2 \left(\frac{L}{R_m}\right)^2}}$$



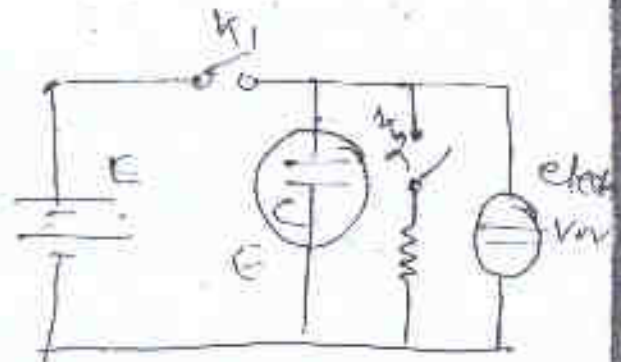
$$\frac{L}{R_{ch}} = \frac{L_{ch}}{R_m}$$

$$\frac{I_{rms}}{I_m} = \frac{\sqrt{R_m^2 + (\omega L)^2}}{\sqrt{R_m^2 + (\omega L_m)^2}}$$

Loss of charge (High)

(i) k_1 closed is

$$i = c \frac{dv}{dt} \text{ (charging)}$$



(ii) k_2 closed

$$i = -c \frac{dv}{dt}$$

$$i = \frac{dq}{dt} \quad i = \frac{V}{R}$$

$$\frac{V}{R} = -c \frac{dv}{dt}$$

$$\Rightarrow -\frac{1}{Rc} \int dt = \int \frac{dv}{v}$$

$$\Rightarrow \log_e v = -\frac{1}{Rc} t + \log_e k$$

$$\text{At } t=0, E = v$$

$$\Rightarrow \log_e E = -\frac{1}{Rc} \times 0 + \log_e k$$

$$\Rightarrow k = E$$

$$\Rightarrow \log_e v = -\frac{1}{Rc} t + \log_e E$$

$$\Rightarrow \frac{1}{Rc} t = \log_e \left(\frac{E}{v} \right)$$

$$\Rightarrow \frac{Rc}{t} = \frac{1}{\log_e \frac{E}{v}}$$

$$\Rightarrow R = \frac{t}{c \log_e \frac{E}{v}}$$

leaf
no.

